CHAPTER 5

APPLICATIONS OF NEWTON'S LAWS

In Chapter 3 we introduced Newton's laws and gave some examples of their applications. Those examples were deliberately oversimplified, so that the use of the laws could be illustrated. In the process of oversimplification, we lost some of the physical insight.

In this chapter we continue with further applications of Newton's laws, particularly to friction and other contact forces, to circular motion, and to nonconstant forces. Finally, we show how using a noninertial reference frame produces effects that can be analyzed by introducing inertial forces or pseudoforces that, in contrast to real forces, are not caused by specific objects in the environment.

5-1 FORCE LAWS

Before we return to applications of Newton's laws, we should briefly discuss the nature of the forces themselves. We have used the equations of motion to analyze and calculate the *effects* of forces, but they tell us nothing about the *causes* of the forces. To understand what causes a force we must have a detailed microscopic understanding of the interactions of objects with their environment. On the most fundamental level, nature appears to operate through a small number of fundamental forces. Physicists have traditionally identified four basic forces: (1) *the gravitational force,* which originates with the presence of matter; (2) *the electromagnetic force,* which includes basic electric and magnetic interactions and is responsible for the binding of atoms and the structure of solids; (3) *the weak nuclear force,* which causes certain radioactive decay processes and certain reactions among the fundamental particles; and (4) *the strong force,* which operates among the fundamental particles and is responsible for binding the nucleus together.

On the most microscopic scale—for example, two protons in a typical nucleus—the relative strengths of these forces would be: strong (relative strength = 1); electromagnetic (10^{-2}); weak (10^{-9}); gravitational (10^{-38}). On the fundamental scale, gravity is exceedingly weak and has negligible effects. You can get some appreciation for the weakness of gravity from some common experiments—for example, lifting a few bits of paper with an electrostatically charged comb or lifting a few nails or paper clips with a magnet. The magnetic force of a small magnet is sufficient to overcome the gravitational force exerted by the entire Earth on these objects!

The search for ever more simplification has led physicists to try to reduce the number of forces even below four. In 1967, a theory was proposed according to which the weak and electromagnetic forces could be regarded as parts of a single force, called the *electroweak* force. The combination or *unification* of these two forces is similar to the 19th-century unification of the separate electric and magnetic forces into a single electromagnetic force. Other new theories, called *grand unification theories,* have been proposed that combine the strong and electroweak forces into a single framework, and there are even "theories of everything," which attempt to include gravity as well.

Fortunately, our analysis of mechanical systems need not invoke such theories. In fact, everything we study about ordinary mechanical systems involves only two forces: gravity and electromagnetism. The gravitational force is apparent in the Earth's attraction for objects, which gives them their weight. The much weaker gravitational attraction of one laboratory object for another is almost always negligible.

All the other forces we normally consider are ultimately electromagnetic in origin: contact forces, such as the nor-

mal force exerted when one object pushes on another and the frictional force produced when one surface rubs against another; viscous forces, such as air resistance; tensile forces, such as in a stretched rope or string; elastic forces, as in a spring; and many others. Microscopically, these forces originate with the forces exerted by one atom on another. Fortunately, when we deal with ordinary mechanical systems we can ignore the microscopic basis and replace the complicated substructure with a single effective force of a specified magnitude and direction.

5-2 TENSION AND NORMAL FORCES*

Figure 5-1*a* shows a worker pulling with a force $\vec{\mathbf{P}}$ on a string that is attached to a crate, accelerating it over the surface that we assume to be frictionless. The force on the crate is exerted not directly by the worker, but rather by the string. We call this force a *tension* $\vec{\mathbf{T}}$.

Partial free-body diagrams (including only horizontal forces) for the string and crate are shown in Fig. 5-1*b*. The string pulls on the crate with tension $\vec{\mathbf{T}}$, and therefore by Newton's third law the crate must pull on the string with a force equal in magnitude to $\vec{\mathbf{T}}$ but in the opposite direction. We assume that the string is very thin, so the tension force is always along the direction of the string. Furthermore, we assume that the string is of negligible mass.

Choosing the x axis to be horizontal with its positive direction to the right in Fig. 5-1, we find the net force on the string in the x direction to be $\Sigma F_x = P - T$. (Here P and T represent respectively the magnitudes of the forces $\vec{\mathbf{P}}$ and $\vec{\mathbf{T}}$.) Newton's second law in the form $\Sigma F_x = ma_x$ then gives $P - T = m_{\text{string}} a_x = 0$, because we have assumed that the mass of the string is zero. From this we conclude that $P = T$.

The net force on the crate in the x direction is $\Sigma F_x = T$, and Newton's second law gives $T = m_{\text{crate}} a_x$. Thus $a_x = T/m_{\text{crate}} = P/m_{\text{crate}}$. The thin, massless string simply transmits the applied force from one end to the other with no change in direction or magnitude—that is, the force $\vec{\mathbf{P}}$ that the worker applies to the string is equal to the force that the string applies to the crate.

The ideal string likewise does not stretch. Suppose we add another crate to our system, creating the configuration shown in Fig. 5-2*a*. As before, the magnitude of the tension $\vec{\mathbf{T}}_1$ in the first string is equal to P. Again choosing the x axis to be horizontal with positive to the right, we can find the x component of the net force on crate 1 to be $\Sigma F_x = T_1 - T_2 = P - T_2$ and, similarly for crate 2, $\Sigma F_x = T_2$. Applying Newton's second law gives:

*To simplify the notation in this chapter, we will no longer label each force with subscripts indicating the body that the force acts on and the body that causes the force. However, as you study the examples in this chapter and solve the problems, you should continue to identify these two bodies for each force that acts.

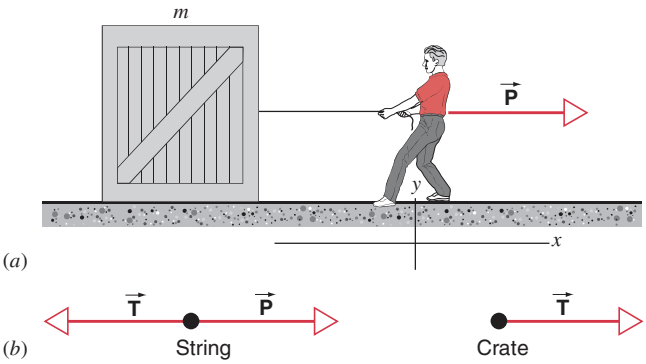


FIGURE 5-1. (*a*) A worker pulls with force P on a string attached to a crate. (*b*) Partial free-body diagrams of the string and the crate, showing only the horizontal forces.

$$\text{crate 1:} \qquad P - T_2 = m_1 a_{1x} \tag{5-1}$$

$$\text{crate 2:} \qquad T_2 = m_2 a_{2x} \tag{5-2}$$

If the second string (which connects m_1 and m_2) does not stretch, then m_1 and m_2 move with the same velocity and acceleration. Putting $a_{1x} = a_{2x} = a_x$, we can combine Eqs. 5-1 and 5-2 to find

$$a_x = \frac{P}{m_1 + m_2} \qquad \text{and} \qquad T_2 = \frac{m_2}{m_1 + m_2} P. \tag{5-3}$$

That is, the two crates accelerate just like a single system of mass $m_1 + m_2$ to which the force $\vec{\mathbf{P}}$ is applied. Considering only the effort exerted by the worker, we could replace the two crates by a single crate of mass $m_1 + m_2$.

The tension force arises because each small element of the string pulls on the element next to it (and is in turn pulled by that element, according to Newton's third law). In this way, a force pulling on one end of the string is transmitted to an object at the other end. This force is due to the force between atoms and is ultimately electromagnetic in origin.

As shown in Fig. 5-3*a*, any particular element i of the string experiences a tension $\vec{\mathbf{T}}$ acting in one direction due to element $i - 1$, and an equal tension acting in the opposite direction due to element $i + 1$. If we were to cut the string at any point and tie a spring scale between the cut ends, the

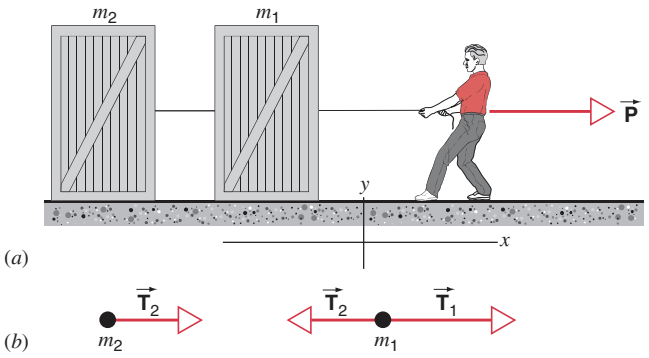


FIGURE 5-2. (*a*) A worker pulls on a string attached to a row of two crates. (*b*) Partial free-body diagrams of the two crates, showing only the horizontal forces.

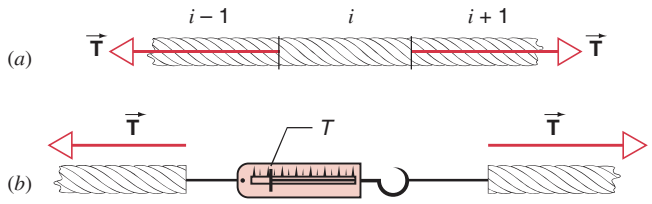


FIGURE 5-3. (*a*) Three small elements of a stretched string, labeled $i - 1$, i, and $i + 1$. The forces acting on element i are shown. (*b*) If the string is cut so that element i is replaced by a spring scale (the rest of the string being undisturbed), the scale reads the tension T.

spring scale would read the magnitude of the tension T directly (Fig. 5-3*b*).

Note that the spring scale does not read $2T$, even though a tension $\vec{\mathbf{T}}$ pulls in each direction on the scale. In the same way, when we hang an object of weight W from a spring scale, the scale reads W and not $2W$, even though there is a downward force W on the spring scale from the weight of the object and also an upward force equal to W on the top end of the scale due to whatever is supporting the scale.

SAMPLE PROBLEM 5-1. Figure 5-4*a* shows a block of mass $m = 15.0$ kg hanging from three strings. What are the tensions in the three strings?

Solution We first consider the knot at the junction of the three strings to be the "body." Figure 5-4*b* shows the free-body diagram of the knot, which is at rest under the action of the three forces $\vec{\mathbf{T}}_A$, $\vec{\mathbf{T}}_B$, and $\vec{\mathbf{T}}_C$, which are due to the tensions in the strings. Choosing the x and y axes as shown, we can resolve the forces into their x and y components, as shown in Fig. 5-4*c*. The acceleration components are zero, so Newton's second law applied to the knot gives

$$x \text{ component: } \sum F_x = -T_A \cos 30° + T_B \cos 45° = ma_x = 0$$

$$y \text{ component: } \sum F_y = T_A \sin 30° + T_B \sin 45° - T_C = ma_y = 0.$$

Figure 5-4*d* shows the free-body diagram of the block. The forces have only y components, and once again the acceleration is zero, so

$$\sum F_y = T_C - mg = ma_y = 0.$$

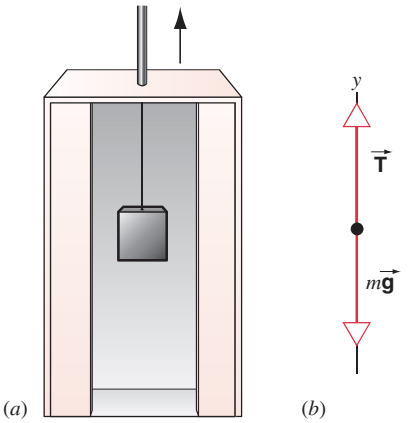


FIGURE 5-5. Sample Problem 5-2. (*a*) A package hangs from a string in an ascending elevator. (*b*) The free-body diagram of the package.

Solving for T_C, we find

$$T_C = mg = (15.0 \text{ kg})(9.80 \text{ m/s}^2) = 147 \text{ N}.$$

Substituting this result, we can solve the two equations for the forces on the knot simultaneously to find

$$T_A = 108 \text{ N} \quad \text{and} \quad T_B = 132 \text{ N}.$$

Check these results to see whether the vector sum of the three forces acting on the knot is indeed zero.

SAMPLE PROBLEM 5-2. A package (mass 2.4 kg) tied to a string hangs from the ceiling of an elevator (Fig. 5-5*a*). What is the tension in the string when the elevator is (*a*) descending with constant velocity, and (*b*) ascending with an acceleration of 3.2 m/s^2?

Solution (*a*) The free-body diagram of the package is shown in Fig. 5-5*b*. Two forces act on the package: the upward force due to the tension in the string and the downward force of the Earth's gravity. We choose the y axis to be vertical and positive upward. The net force on the package is $\Sigma F_y = T - mg$. Newton's second law ($\Sigma F_y = ma_y$) then gives $T - mg = ma_y$ or, solving for the tension T,

$$T = m(g + a_y).$$

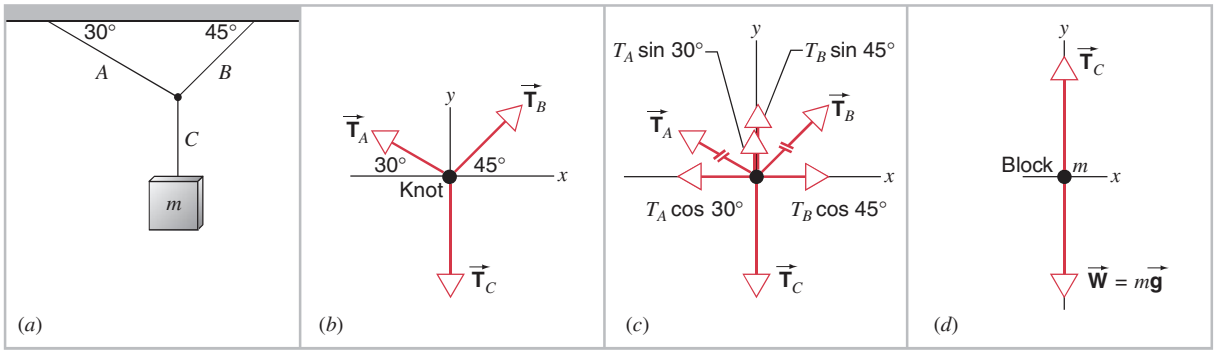


FIGURE 5-4. Sample Problem 5-1. (*a*) A block hangs from three strings A, B, and C. (*b*) The free-body diagram of the knot that joins the strings. (*c*) The free-body diagram of the knot, with $\vec{\mathbf{T}}_A$ and $\vec{\mathbf{T}}_B$ resolved into their x and y vector components. The double lines on a vector remind us that we have replaced the vector by its vector components. (*d*) The free-body diagram of the block.

When the elevator moves with constant velocity, $a_y = 0$ and so

$$T = mg = (2.4\text{ kg})(9.8\text{ m/s}^2) = 24\text{ N}.$$

(*b*) When the elevator moves with $a_y = +3.2\text{ m/s}^2$, the tension is

$$T = m(g + a_y) = (2.4\text{ kg})(9.8\text{ m/s}^2 + 3.2\text{ m/s}^2) = 31\text{ N}.$$

In this case the elevator is moving upward and increasing its speed. Would you expect the tension to be greater than it is when the elevator is at rest or moving with constant velocity? Suppose the elevator were moving downward and braking, so that its acceleration is upward and again equal to $+3.2\text{ m/s}^2$. Would the tension in the string have the same value? Is this reasonable? What would be the tension if the elevator were in free fall?

Compare this problem with Sample Problem 3-7 and account for any similarities or differences.

The Normal Force

Consider a book resting on a table, as shown in Fig. 5-6*a*. Gravity exerts a downward force on the book, but the book has no vertical acceleration. The net vertical force on the book must therefore be zero, so there must be an additional upward force acting on it. This force is the *normal* force exerted on the book by the table. In this sense, the word "normal" means "perpendicular"—the normal force exerted by a surface is *always* perpendicular to (or normal to) the surface.

Even though the normal force shown in the free-body diagram of Fig. 5-6*b* is equal and opposite to the weight, it is *not* the reaction force to the weight. The weight is the force of the Earth on the book, and its reaction force is the force exerted by the book on the Earth. The reaction force to the normal force is the downward force exerted by the book on the table; it would appear in a free-body diagram of the table. Remember that the action–reaction pairs of Newton's third law *never* act on the same body, so the forces N and W that act on the book cannot be an action–reaction pair.

If someone placed a hand on top of the book and pushed downward with a force P, the book would remain at rest. For an acceleration of zero, the net force on the book must be zero and so the total downward force $W + P$ must equal the total upward force N. The normal force must therefore increase as P increases, since $N = W + P$. Eventually, P could become large enough to exceed the ability of the table to provide the upward normal force, and the book would break through the tabletop.

Tension and normal forces are examples of *contact* forces, in which one body exerts a force on another because of the contact between them. These forces originate with the atoms of each body—each atom exerts a force on its neighbor (which may be an atom of another body). A contact force can be maintained only if it does not exceed the interatomic forces within either of the bodies; otherwise the binding between atoms can be overcome and the string or the surface will split into pieces.

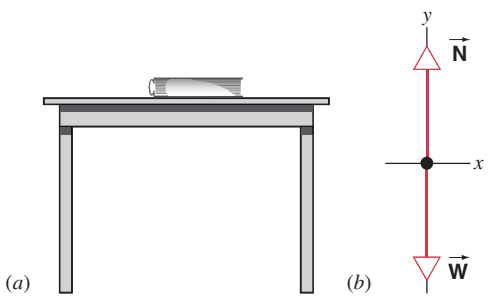


FIGURE 5-6. (*a*) A book resting on a table. (*b*) The free-body diagram of the book.

SAMPLE PROBLEM 5-3. A sled of mass $m = 7.5$ kg is pulled along a frictionless horizontal surface by a cord (Fig. 5-7*a*). A constant force of $P = 21.0$ N is applied to the cord. Analyze the motion if (*a*) the cord is horizontal and (*b*) the cord makes an angle of $\theta = 15°$ with the horizontal.

Solution (*a*) The free-body diagram with the cord horizontal is shown in Fig. 5-7*b*. The surface exerts a force N, the normal force, on the sled. The components of the net force acting on the sled are $\Sigma F_x = P$ and $\Sigma F_y = N - mg$, and applying Newton's second law we obtain

x component ($\Sigma F_x = ma_x$): $\quad P = ma_x$

y component ($\Sigma F_y = ma_y$): $\quad N - mg = ma_y$

If there is to be no vertical motion, the sled remains on the surface and $a_y = 0$. Thus

$$N = mg = (7.5\text{ kg})(9.80\text{ m/s}^2) = 74\text{ N}.$$

The horizontal acceleration is

$$a_x = \frac{P}{m} = \frac{21.0\text{ N}}{7.5\text{ kg}} = 2.80\text{ m/s}^2.$$

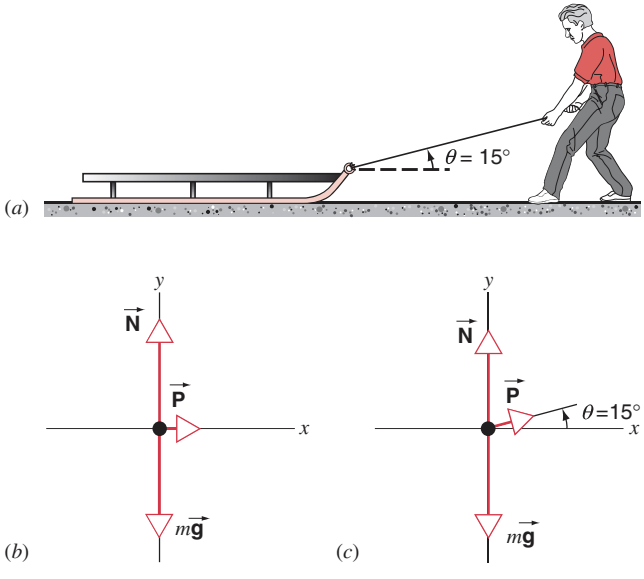


FIGURE 5-7. Sample Problem 5-3. (*a*) A sled is pulled along a frictionless horizontal surface. (*b*) The free-body diagram of the sled when $\theta = 0°$. (*c*) The free-body diagram of the sled when $\theta = 15°$.

Note that, if the surface is truly frictionless, as we have assumed, the person would find it difficult to continue to exert this force on the sled for very long. After 30 s at this acceleration, the sled would be moving at 84 m/s or 188 mi/h!

(*b*) When the pulling force is not horizontal, the free-body diagram is shown in Fig. 5-7*c*, and the components of the net force are $\Sigma F_x = P\cos\theta$ and $\Sigma F_y = N + P\sin\theta - mg$. Newton's second law then gives

x component ($\Sigma F_x = ma_x$): $\quad P\cos\theta = ma_x$

y component ($\Sigma F_y = ma_y$): $\quad N + P\sin\theta - mg = ma_y$

Let us assume for the moment that the sled stays on the surface; that is, $a_y = 0$. Then

$$N = mg - P\sin\theta = 74\text{ N} - (21.0\text{ N})(\sin 15°) = 69\text{ N},$$

$$a_x = \frac{P\cos\theta}{m} = \frac{(21.0\text{ N})(\cos 15°)}{7.5\text{ kg}} = 2.70\text{ m/s}^2.$$

A normal force is always perpendicular to the surface in contact; with the coordinates chosen as in Fig. 5-7*b*, N must be positive. If we increase $P\sin\theta$, N would decrease and at some point would become zero. At that point the sled would leave the surface, under the influence of the upward component of P, and we would need to analyze its vertical motion. With the values of P and θ we have used, the sled remains on the surface and $a_y = 0$.

Note that a_x is smaller in part (*b*) than in part (*a*). Can you explain this?

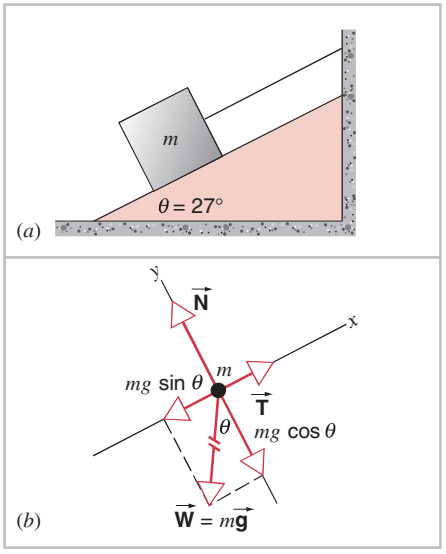


FIGURE 5-8. Sample Problem 5-4. (*a*) A mass m is supported at rest by a string on a frictionless inclined plane. (*b*) The free-body diagram of m. Note that the xy coordinate system is tilted so that the x axis is parallel to the plane. The weight $m\vec{\mathbf{g}}$ has been resolved into its vector components; the double line through the vector $m\vec{\mathbf{g}}$ reminds us that this vector has been replaced by its components.

SAMPLE PROBLEM 5-4. A block of mass $m = 18.0$ kg is held in place by a string on a frictionless plane inclined at an angle of 27° (see Fig. 5-8*a*). (*a*) Find the tension in the string and the normal force exerted on the block by the plane. (*b*) Analyze the subsequent motion after the string is cut.

Solution (*a*) The free-body diagram of the block is shown in Fig. 5-8*b*. The block is acted on by the normal force $\vec{\mathbf{N}}$, its weight $\vec{\mathbf{W}} = m\vec{\mathbf{g}}$ and a force due to the tension $\vec{\mathbf{T}}$ of the string. We choose a coordinate system with the x axis along the plane and the y axis perpendicular to it. With this choice, two of the forces ($\vec{\mathbf{T}}$ and $\vec{\mathbf{N}}$) are already resolved into components, and the motion that will eventually occur along the plane has only one component as well. The weight is resolved into its x component $-mg\sin\theta$ and its y component $-mg\cos\theta$. The net force in the x direction is then $\Sigma F_x = T - mg\sin\theta$ and the net force in the y direction is $\Sigma F_y = N - mg\cos\theta$.

In the static case, $a_x = 0$ and $a_y = 0$. Newton's second law then gives $\Sigma F_x = ma_x = 0$ and $\Sigma F_y = ma_y = 0$, so

$$T - mg\sin\theta = 0 \qquad \text{and} \qquad N - mg\cos\theta = 0.$$

Examine these equations. Are they reasonable? What happens in the limit $\theta = 0°$? It looks like the tension would be zero. Would you expect the tension to be zero if the block were resting on a horizontal surface? What happens to the normal force when $\theta = 0°$? Is this reasonable? What happens to T and N in the limit of $\theta = 90°$? You should form the habit of asking questions like these before starting on the algebra to find the solution. If there is an error, now is the best time to find and correct it.

Solving the equations,

$$T = mg\sin\theta = (18.0\text{ kg})(9.80\text{ m/s}^2)(\sin 27°) = 80\text{ N},$$

$$N = mg\cos\theta = (18.0\text{ kg})(9.80\text{ m/s}^2)(\cos 27°) = 157\text{ N}.$$

(*b*) When the string is cut, the tension disappears from the equations and the block is no longer in equilibrium. The components of the net force are now $\Sigma F_x = -mg\sin\theta$ and $\Sigma F_y = N - mg\cos\theta$. Newton's second law for the x and y components now gives

$$-mg\sin\theta = ma_x \qquad \text{and} \qquad N - mg\cos\theta = ma_y.$$

Cutting the string doesn't change the motion in the y direction (the block doesn't jump off the plane!), so $a_y = 0$ as before and the normal force still equals $mg\cos\theta$, or 157 N. In the x direction

$$a_x = -g\sin\theta = -(9.80\text{ m/s}^2)(\sin 27°) = -4.45\text{ m/s}^2.$$

The negative sign shows that the block accelerates in the negative x direction—that is, down the plane. Check the limits $\theta = 0°$ and $\theta = 90°$. Are they consistent with your expectations?

Additional Applications

Here we consider some additional applications of Newton's laws. These examples involve two objects that must be analyzed separately but not quite independently, because the motion of one object is constrained by the motion of another, such as when they are attached to one another by a string of fixed length. Study these examples, and note the independent choices of coordinate systems used for the separate objects.

SAMPLE PROBLEM 5-5. Two blocks with unequal masses m_1 and m_2 are connected by a string that passes over an ideal pulley (whose mass is negligible and that rotates with negligible friction), as shown in Fig. 5-9. (The arrangement is also known as an

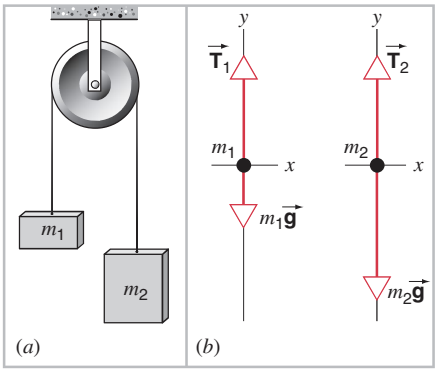


FIGURE 5-9. Sample Problem 5-5. (*a*) Diagram of Atwood's machine, consisting of two suspended blocks connected by a string that passes over a pulley. (*b*) Free-body diagrams of m_1 and m_2.

*Atwood's machine.**) Let m_2 be greater than m_1. Find the tension in the string and the acceleration of the blocks.

Solution We choose our coordinate system with the positive y axis upward; only y components of forces and accelerations need be considered. The free-body diagrams are shown in Fig. 5-9*b*. For m_1 the net force is $\Sigma F_y = T_1 - m_1 g$; for m_2, $\Sigma F_y = T_2 - m_2 g$. Applying Newton's second law in the y direction for each block gives:

block 1: $$T_1 - m_1 g = m_1 a_{1y}$$

block 2: $$T_2 - m_2 g = m_2 a_{2y}$$

where a_{1y} and a_{2y} are the respective accelerations of m_1 and m_2. If the string is massless and doesn't stretch, and if the pulley is massless and frictionless, then the tension has the same magnitude everywhere in the string and the magnitudes of the accelerations of the blocks are equal. (This ideal pulley doesn't change the magnitude of the tension or the acceleration from one side of the string to the other; its only function is to change their directions.) We set $T_1 = T_2 = T$, the common value of the tension in the string. If we let a represent the common magnitude of the accelerations, then $a_{1y} = a$ (a positive number, because the less massive block 1 moves upward) and $a_{2y} = -a$ (a negative number, because the more massive block 2 moves downward). Making these substitutions and solving the two equations simultaneously, we find

$$a = \frac{m_2 - m_1}{m_2 + m_1} g \qquad \text{and} \qquad T = \frac{2 m_1 m_2}{m_1 + m_2} g. \tag{5-4}$$

Consider what happens in the limiting cases $m_1 = 0$, $m_2 = 0$, $g = 0$, and $m_1 = m_2$. Note that $m_1 g < T < m_2 g$, and be sure you understand why this must be true. If the system is accelerating, then the tension in the string is *not* equal to the weight of either block.

*George Atwood (1745–1807) was an English mathematician who developed this device in 1784 for demonstrating the laws of accelerated motion and measuring g. By making the difference between m_1 and m_2 small, he was able to "slow down" the effect of free fall and time the motion of the falling weight with a pendulum clock, the most precise way to measure time intervals in his day.

SAMPLE PROBLEM 5-6. Figure 5-10*a* shows a block of mass m_1 on a frictionless horizontal surface. The block is pulled by a string of negligible mass that is attached to a hanging block of mass m_2. The string passes over a pulley whose mass is negligible and whose axle rotates with negligible friction. Find the tension in the string and the acceleration of each block.

Solution Figures 5-10*b* and 5-10*c* show the free-body diagrams for the two blocks.

Block 1 is acted on by a normal force due to the surface, by gravity, and by a force due to the tension in the string. The components of the net force on block 1 are $\Sigma F_x = T_1$ and $\Sigma F_y = N - m_1 g$, and Newton's second law for block 1 then gives:

$$T_1 = m_1 a_{1x} \qquad \text{and} \qquad N - m_1 g = m_1 a_{1y}.$$

We expect that block 1 does not move in the y direction, so that $a_{1y} = 0$.

For block 2, there are no forces in the x direction. The net force in the y direction is $\Sigma F_y = T_2 - m_2 g$, and Newton's second law gives

$$T_2 - m_2 g = m_2 a_{2y}.$$

If the string is of negligible mass and the pulley is ideal (frictionless and of negligible mass), then the magnitudes of the tension forces T_1 and T_2 are equal; we let T represent the common value of the tension. If the string doesn't stretch, then the accelerations of the blocks are equal; letting a be the common value of the acceleration, we set $a_{1x} = a$ and $a_{2y} = -a$. We now have two equations:

$$T = m_1 a \qquad \text{and} \qquad T - m_2 g = m_2(-a).$$

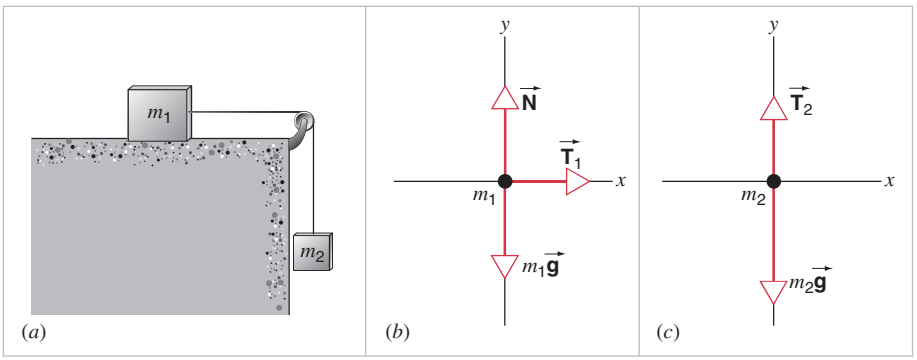


FIGURE 5-10. Sample Problem 5-6. (*a*) Block m_1 is pulled along a smooth horizontal surface by a string that passes over a pulley and is attached to block m_2. (*b*) The free-body diagram of block m_1. (*c*) The free-body diagram of block m_2.

Solving these simultaneously, we obtain

$$a = \frac{m_2}{m_1 + m_2} g \qquad \text{and} \qquad T = \frac{m_1 m_2}{m_1 + m_2} g. \qquad (5\text{-}5)$$

It is helpful to consider the limiting cases of these results. What happens when m_1 is zero? We would expect the string to be slack ($T = 0$) and m_2 to be in free fall ($a = g$). The equations correctly predict these limits. When $m_2 = 0$, there is no horizontal force on block 1 and it does not accelerate; again, the equations give the correct prediction.

Note that $a < g$, as we should expect. Also, note that T is less than mg, as we should expect when the block is accelerating downward (see Sample Problem 5-2).

Do Eqs. 5-5 behave properly in the limit $g = 0$?

SAMPLE PROBLEM 5-7. In the system shown in Fig. 5-11*a*, a block (of mass $m_1 = 9.5$ kg) slides on a frictionless plane inclined at an angle $\theta = 34°$. The block is attached by a string to a second block (of mass $m_2 = 2.6$ kg). The system is released from rest. Find the acceleration of the blocks and the tension in the string.

Solution The free-body diagrams for blocks 1 and 2 are shown in Figs. 5-11*b* and 5-11*c*. We choose coordinate systems as shown, so that one coordinate axis is parallel to the anticipated acceleration of each body. As in the previous examples, we expect that the tension has a common value and that the vertical motion of m_2 and the motion along the plane of m_1 can be described by accelerations of the same magnitude. We arbitrarily assume that m_1 moves in the positive x direction (if our assumption is wrong, a will come out negative). The components of the net force on m_1 are $\Sigma F_x = T - m_1 g \sin\theta$ and $\Sigma F_y = N - m_1 g \cos\theta$, and Newton's second law gives (with $a_{1x} = a$ and $a_{1y} = 0$):

$$T - m_1 g \sin\theta = m_1 a \qquad \text{and} \qquad N - m_1 g \cos\theta = 0.$$

For m_2 the y component of the net force is $\Sigma F_y = T - m_2 g$, and Newton's second law is (with $a_{2y} = -a$)

$$T - m_2 g = m_2(-a).$$

Solving simultaneously gives

$$a = \frac{m_2 - m_1 \sin\theta}{m_1 + m_2} g \qquad (5\text{-}6a)$$

and

$$T = \frac{m_1 m_2 g}{m_1 + m_2}(1 + \sin\theta). \qquad (5\text{-}6b)$$

Note that these results duplicate Eqs. 5-5 of Sample Problem 5-6 if we put $\theta = 0$ (so that block 1 moves horizontally) and Eqs. 5-4 of Sample Problem 5-5 if we put $\theta = 90°$ (so that block 1 moves vertically).

Putting in the numbers, we have

$$a = \frac{2.6\text{ kg} - (9.5\text{ kg})(\sin 34°)}{9.5\text{ kg} + 2.6\text{ kg}}(9.80\text{ m/s}^2) = -2.2\text{ m/s}^2.$$

The acceleration comes out to be negative, which means our initial guess about the direction of motion was wrong. Block 1 slides down the plane, and block 2 moves upward. Because the dynamical equations do not involve forces that depend on the direction of motion, this incorrect initial guess has no effect on the equations and we can accept the final value as correct. In general, this will not be the case when we consider frictional forces that act opposite to the direction of motion.

For the tension in the string, we find

$$T = \frac{(9.5\text{ kg})(2.6\text{ kg})(9.80\text{ m/s}^2)}{9.5\text{ kg} + 2.6\text{ kg}}(1 + \sin 34°) = 31\text{ N}.$$

This value is greater than the weight of m_2 ($m_2 g = 26$ N), which is consistent with the acceleration of m_2 being upward.

5-3 FRICTIONAL FORCES*

A block of mass m moving with initial velocity $\vec{\mathbf{v}}_0$ along a long horizontal table will eventually come to rest. This means that, while it is moving, it experiences an average acceleration $\vec{\mathbf{a}}_{av}$ that points in the direction opposite to its motion. If (in an inertial frame) we see that a body is accelerated, we always associate a force, defined from Newton's second law, with the motion. In this case we declare that the table exerts a force of *friction,* whose average value is $m\vec{\mathbf{a}}_{av}$, on the sliding block. We generally take friction to mean a contact interaction between solids. Frictionlike effects caused by liquids and gases are described by other terms (see Section 4-4).

Actually, whenever the surface of one body slides over that of another, each body exerts a frictional force on the other. The frictional force on each body is in a direction

*See "Friction at the Atomic Scale" by Jacqueline Krim, *Scientific American,* October 1996, p. 74.

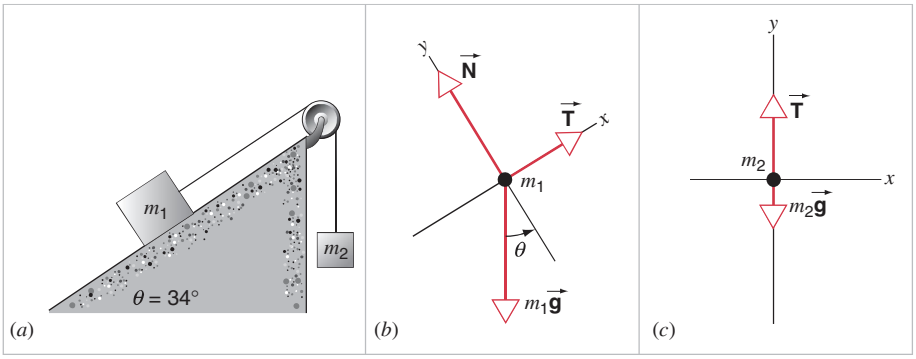


FIGURE 5-11. Sample Problem 5-7. (*a*) Block m_1 slides on a frictionless inclined plane. Block m_2 hangs from a string attached to m_1. (*b*) Free-body diagram of m_1. (*c*) Free-body diagram of m_2.

opposite to its motion relative to the other body. Frictional forces automatically oppose this relative motion and never aid it. Even when there is no relative motion, frictional forces may exist between surfaces.

Although we have ignored its effects up to now, friction is very important in our daily lives. Left to act alone it brings every rotating shaft to a halt. In an automobile, about 20% of the engine power is used to counteract frictional forces. Friction causes wear and seizing of moving parts, and much engineering effort is devoted to reducing it. On the other hand, without friction we could not walk; we could not hold a pencil, and if we could it would not write; wheeled transport as we know it would not be possible.

We want to express frictional forces in terms of the properties of the body and its environment; that is, we want to know the force law for frictional forces. In what follows we consider the sliding (not rolling) of one dry (unlubricated) surface over another. As we shall see later, friction, viewed at the microscopic level, is a very complicated phenomenon. The force laws for dry, sliding friction are empirical in character and approximate in their predictions. They do not have the elegant simplicity and accuracy that we find for the gravitational force law (Chapter 14) or for the electrostatic force law (Chapter 25). It is remarkable, however, considering the enormous diversity of surfaces one encounters, that many aspects of frictional behavior can be understood qualitatively on the basis of a few simple mechanisms.

Consider a block at rest on a horizontal table as in Fig. 5-12*a*. Attach a spring to it to measure the horizontal force $\vec{\mathbf{F}}$ required to set the block in motion. We find that the block will not move even though we apply a small force (Fig. 5-12*b*). We say that our applied force is balanced by an opposite frictional force $\vec{\mathbf{f}}$ exerted on the block by the table, acting along the surface of contact. As we increase the applied force (Fig. 5-12*c, d*) we find some definite force at which the block will "break away" from the surface and begin to accelerate (Fig. 5-12*e*). By reducing the force once motion has started, we find that it is possible to keep the block in uniform motion without acceleration (Fig. 5-12*f*). Figure 5-12*g* shows the results of an experiment to measure the frictional force. An increasing force F is applied starting at about $t = 2$ s, after which the frictional force increases with the applied force and the object remains at rest. At $t = 4$ s, the object suddenly begins to move and the frictional force becomes constant, independent of the applied force.

The frictional forces acting between surfaces at rest with respect to each other are called forces of *static friction.* The maximum force of static friction (corresponding to the peak at $t = 4$ s in Fig. 5-12*g*) will be the same as the smallest applied force necessary to start motion. Once motion is started, the frictional forces acting between the surfaces usually decrease so that a smaller force is necessary to maintain uniform motion (corresponding to the nearly constant force at $t > 4$ s in Fig. 5-12*g*). The forces acting between surfaces in relative motion are called forces of *kinetic friction.*

The maximum force of static friction between any pair of dry unlubricated surfaces follows these two empirical

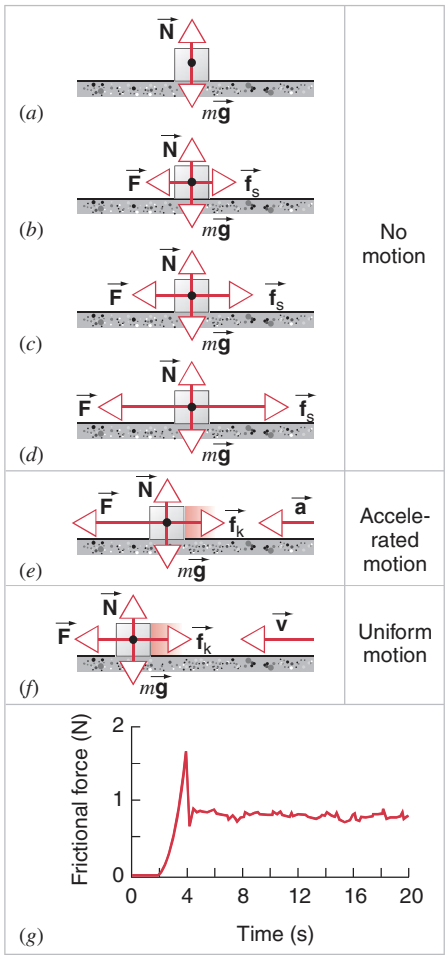


FIGURE 5-12. (*a–d*) An external force $\vec{\mathbf{F}}$, applied to a resting block, is counterbalanced by an equal but opposite frictional force $\vec{\mathbf{f}}$. As $\vec{\mathbf{F}}$ is increased, $\vec{\mathbf{f}}$ also increases, until $\vec{\mathbf{f}}$ reaches a certain maximum value. (*e*) The block then "breaks away," accelerating to the left. (*f*) If the block is to move with constant velocity, the applied force $\vec{\mathbf{F}}$ must be reduced from the maximum value it had just before the block began to move. (*g*) Experimental results; here the applied force $\vec{\mathbf{F}}$ is increased from zero starting at about $t = 2$ s, and the motion suddenly begins at about $t = 4$ s. For details of the experiment, see "Undergraduate Computer-Interfacing Projects," by Joseph Priest and John Snyder, *The Physics Teacher,* May 1987, p. 303.

laws: (1) It is approximately independent of the area of contact over wide limits and (2) it is proportional to the normal force.*

The ratio of the magnitude of the *maximum* force of static friction to the magnitude of the normal force is called the *coefficient of static friction* for the surfaces involved. If

*The two laws of friction were first discovered experimentally by Leonardo da Vinci (1452–1519). Leonardo's statement of the two laws was remarkable, coming as it did two centuries before Newton developed the concept of force. The mathematical expressions of the laws of friction and the concept of the coefficient of friction were developed by Charles Augustin Coulomb (1736–1806), who is best known for his studies of electrostatics (see Chapter 25).

TABLE 5-1 Coefficients of Friction[a]

Surfaces	μ_s	μ_k
Wood on wood	0.25–0.5	0.2
Glass on glass	0.9–1.0	0.4
Steel on steel, clean surfaces	0.6	0.6
Steel on steel, lubricated	0.09	0.05
Rubber on dry concrete	1.0	0.8
Waxed wood ski on dry snow	0.04	0.04
Teflon on Teflon	0.04	0.04

[a]Values are approximate and are intended only as estimates. The actual coefficients of friction for any pair of surfaces depend on such conditions as the cleanliness of the surfaces, the temperature, and the humidity.

f_s represents the magnitude of the force of static friction, we can write

$$f_s \leq \mu_s N, \tag{5-7}$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force. The equality sign holds only when f_s has its maximum value.

The force of kinetic friction f_k between dry, unlubricated surfaces follows the same two laws as those of static friction. (1) It is approximately independent of the area of contact over wide limits and (2) it is proportional to the normal force. The force of kinetic friction is also reasonably independent of the relative speed with which the surfaces move over each other.

The ratio of the magnitude of the force of kinetic friction to the magnitude of the normal force is called the *coefficient of kinetic friction.* If f_k represents the magnitude of the force of kinetic friction, then

$$f_k = \mu_k N, \tag{5-8}$$

where μ_k is the coefficient of kinetic friction.

Both μ_s and μ_k are dimensionless constants, each being the ratio of (the magnitudes of) two forces. Usually, for a given pair of surfaces $\mu_s > \mu_k$. The actual values of μ_s and μ_k depend on the nature of both the surfaces in contact. In most cases we can regard them as being constants (for a given pair of surfaces) over the range of forces and velocities we commonly encounter. Both μ_s and μ_k can exceed unity, although commonly they are less than 1. Table 5-1 shows some representative values of μ_s and μ_k.

FIGURE 5-13. A magnified section of a highly polished steel surface. The vertical scale of the surface irregularities is several thousand atomic diameters. The section has been cut at an angle so that the vertical scale is exaggerated with respect to the horizontal scale by a factor of 10.

Note that Eqs. 5-7 and 5-8 are relations between the *magnitudes only* of the normal and frictional forces. These forces are always directed perpendicularly to one another.

The Microscopic Basis of Friction

On the atomic scale even the most finely polished surface is far from flat. Figure 5-13, for example, shows an actual profile, highly magnified, of a steel surface that would be considered to be highly polished. One can readily believe that when two bodies are placed in contact, the actual microscopic area of contact is much less than the true area of the surface; in a particular case these areas can easily be in the ratio of $1:10^4$.

The actual (microscopic) area of contact is proportional to the normal force, because the contact points deform plastically under the great stresses that develop at these points. Many contact points actually become "cold-welded" together. This phenomenon, *surface adhesion,* occurs because at the contact points the molecules on opposite sides of the surface are so close together that they exert strong intermolecular forces on each other.

When one body (a metal, say) is pulled across another, the frictional resistance is associated with the rupturing of these thousands of tiny welds, which continually reform as new chance contacts are made (see Fig. 5-14). Radioactive tracer experiments have shown that, in the rupturing process, small fragments of one metallic surface may be sheared off and adhere to the other surface. If the relative speed of the two surfaces is great enough, there may be local melting at certain contact areas even though the surface as a whole may feel only moderately warm. The "stick and

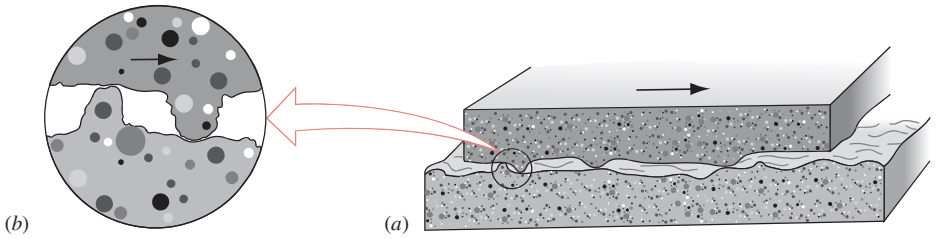


FIGURE 5-14. The mechanism of sliding friction. (*a*) The upper surface is sliding to the right over the lower surface in this enlarged view. (*b*) A detail, showing two spots where cold welding has occurred. Force is required to break these welds and maintain the motion. If the normal force increases, the surfaces are pushed together so that more welds form and the frictional force increases.

slip" events are responsible for the noises that dry surfaces make when sliding across one another as, for example, the squealing chalk on the blackboard.

The coefficient of friction depends on many variables, such as the nature of the materials, surface finish, surface films, temperature, and extent of contamination. For example, if two carefully cleaned metal surfaces are placed in a highly evacuated chamber so that surface oxide films do not form, the coefficient of friction rises to enormous values and the surfaces actually become firmly "welded" together. The admission of a small amount of air to the chamber so that oxide films may form on the opposing surfaces reduces the coefficient of friction to its "normal" value.

The frictional force that opposes one body *rolling* over another is much less than that for sliding motion; this gives the advantage to the rolling wheel over the sliding sledge. This reduced friction is due in large part to the fact that, in rolling, the microscopic contact welds are "peeled" apart rather than "sheared" apart as in sliding friction. This reduces the frictional force by a large factor.

Frictional resistance in dry, sliding friction can be reduced considerably by lubrication. This technique was used in ancient Egypt to move the blocks from which the pyramids were built. A still more effective technique is to introduce a layer of gas between the sliding surfaces; a laboratory air track and the gas-supported bearing are two examples. Friction can be reduced still further by suspending an object by means of magnetic forces. Magnetically levitated trains now under development have the potential for high-speed, nearly frictionless travel.

SAMPLE PROBLEM 5-8. A block is at rest on an inclined plane making an angle θ with the horizontal, as in Fig. 5-15*a*. As the angle of incline is raised, it is found that slipping just begins at an angle of inclination $\theta_s = 15°$. What is the coefficient of static friction between block and incline?

Solution The forces acting on the block, considered to be a particle, are shown in Fig. 5-15*b*. The weight of the block is $m\vec{\mathbf{g}}$, the normal force exerted on the block by the inclined surface is $\vec{\mathbf{N}}$, and the force of friction exerted by the inclined surface on the block is $\vec{\mathbf{f}}_s$. Note that the resultant force exerted by the inclined surface on the block, $\vec{\mathbf{N}} + \vec{\mathbf{f}}_s$, is no longer perpendicular to the surface of contact, as was true for frictionless surfaces ($\vec{\mathbf{f}}_s = 0$). The block is at rest, so that Newton's second law gives $\Sigma\,\vec{\mathbf{F}} = 0$.

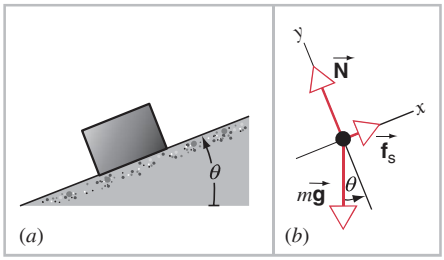


FIGURE 5-15. Sample Problem 5-8. (*a*) A block at rest on an inclined plane. (*b*) A free-body diagram of the block.

Resolving the weight into its x and y components (see Fig. 5-8), we can find the components of the net force to be $\Sigma F_x = f_s - mg \sin\theta$ and $\Sigma F_y = N - mg\cos\theta$. If the block is at rest, then $a_x = 0$ and $a_y = 0$, and Newton's second law gives

$$f_s - mg\sin\theta = 0 \qquad \text{and} \qquad N - mg\cos\theta = 0.$$

At the angle θ_s where slipping just begins, f_s has its maximum value of $\mu_s N$. Evaluating f_s and N from the above equations, we then have

$$\mu_s = \frac{f_s}{N} = \frac{mg\sin\theta_s}{mg\cos\theta_s} = \tan\theta_s = \tan 15° = 0.27.$$

Hence measurement of the angle of inclination at which slipping just starts provides a simple experimental method for determining the coefficient of static friction between two surfaces. Note that this determination is independent of the weight of the object.

You can use similar arguments to show that the angle of inclination θ_k required to maintain a *constant speed* for the block as it slides down the plane, once it has been started by a gentle tap, is given by

$$\mu_k = \tan\theta_k,$$

where $\theta_k < \theta_s$. With the aid of a ruler to measure the tangent of the angle of inclination, you can now determine μ_s and μ_k for a coin sliding down your textbook.

SAMPLE PROBLEM 5-9. Consider an automobile moving along a straight horizontal road with a speed v_0. The driver applies the brakes and brings the car to a halt without skidding. If the coefficient of static friction between the tires and the road is μ_s, what is the shortest distance in which the automobile can be stopped?

Solution The forces acting on the automobile are shown in Fig. 5-16. The car is assumed to be moving in the positive x direction. If we assume that f_s is a constant force, we have uniformly decelerated motion.

Our plan is to use Newton's laws to find the acceleration of the automobile, and then to use the equations of kinematics from Chapter 2 to find the stopping distance. From the free-body diagram of Fig. 5-16*b*, we obtain the component equations for the net force, $\Sigma F_x = -f_s$ and $\Sigma F_y = N - mg$, and thus for Newton's second law

$$-f_s = ma_x \qquad \text{and} \qquad N - mg = ma_y = 0,$$

where we have put $a_y = 0$ because the car does not move in the vertical direction. From these equations and the force of static friction ($f_s = \mu_s N$) we obtain

$$a_x = -\frac{f_s}{m} = -\frac{\mu_s N}{m} = -\frac{\mu_s(mg)}{m} = -\mu_s g.$$

If the car starts with velocity v_{0x} and ends with velocity $v_x = 0$, we can use Eq. 2-26 ($v_x = v_{0x} + a_x t$) to find the stopping time $t = -v_{0x}/a_x = v_0/\mu_s g$. The stopping distance $d = x - x_0$ can then be found from Eq. 2-28 ($x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$) using this value for t:

$$d = v_{0x}t + \tfrac{1}{2}a_x t^2 = v_0\left(\frac{v_0}{\mu_s g}\right) + \tfrac{1}{2}(-\mu_s g)\left(\frac{v_0}{\mu_s g}\right)^2 = \frac{v_0^2}{2\mu_s g}.$$

The greater is the initial speed, the longer is the distance required to come to a stop; in fact, this distance varies as the square

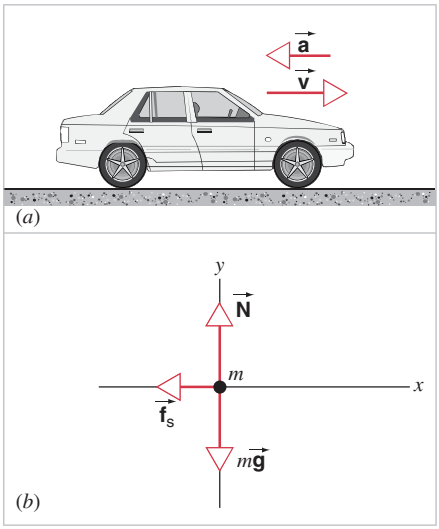


FIGURE 5-16. Sample Problem 5-9. (*a*) A decelerating automobile. (*b*) A free-body diagram of the decelerating automobile, considered to be a particle. For convenience, all forces are taken to act at a common point. In reality, the forces $\vec{\mathbf{N}}$ and $\vec{\mathbf{f}}_s$ are sums of the individual forces exerted by the road on each of the four tires.

of the initial velocity. Also, the greater is the coefficient of static friction between the surfaces, the less is the distance required to come to a stop.

We have used the coefficient of static friction in this problem, rather than the coefficient of kinetic friction, because we assume there is no sliding between the tires and the road. Furthermore, we have assumed that the maximum force of static friction ($f_s = \mu_s N$) operates because the problem seeks the shortest distance for stopping. With a smaller static frictional force the distance for stopping would obviously be greater. The correct braking technique required here is to keep the car just on the verge of skidding. (Cars equipped with anti-lock braking systems maintain this condition automatically). If the surface is smooth and the brakes are fully applied, sliding may occur. In this case μ_k replaces μ_s, and the distance required to stop would increase because μ_k is smaller than μ_s.

As a specific example, if $v_0 = 60$ mi/h $= 27$ m/s, and $\mu_s = 0.60$ (a typical value), we obtain

$$d = \frac{v_0^2}{2\mu_s g} = \frac{(27\ \text{m/s})^2}{2(0.60)(9.8\ \text{m/s}^2)} = 62\ \text{m}.$$

Note that this result is independent of the mass of the car. On rear-wheel drive cars, with the engine in front, it is a common practice to "weigh down" the trunk in order to increase safety when driving on icy roads. How can this practice be consistent with our result that the stopping distance is independent of the mass of the car? (*Hint:* See Exercise 10.)

SAMPLE PROBLEM 5-10. Repeat Sample Problem 5-7, taking into account a frictional force between block 1 and the plane. Use the values $\mu_s = 0.24$ and $\mu_k = 0.15$.

Solution As in Sample Problem 5-7, we assume that block 1 moves down the plane, and so the frictional force acts up the plane. Figure 5-17*a* shows the free-body diagram for m_1. Resolving the forces on m_1 into their components, we have $\Sigma F_x = T + f - m_1 g \sin\theta$ and $\Sigma F_y = N - m_1 g \cos\theta$, and Newton's second law gives

$$T + f - m_1 g \sin\theta = m_1 a \qquad \text{and} \qquad N - m_1 g \cos\theta = 0,$$

where we have put $a_{1x} = a$ and $a_{1y} = 0$, as in Sample Problem 5-7. The net force on block 2 is $\Sigma F_y = T - m_2 g$, and with $a_{2y} = -a$ Newton's second law gives

$$T - m_2 g = m_2(-a).$$

Putting $f = \mu_k N = \mu_k m_1 g \cos\theta$ and solving the remaining two equations for a and T, we obtain

$$a = \frac{m_2 - m_1(\sin\theta - \mu_k \cos\theta)}{m_1 + m_2} g, \qquad (5\text{-}9a)$$

$$T = \frac{m_1 m_2 g}{m_1 + m_2}(1 + \sin\theta - \mu_k \cos\theta). \qquad (5\text{-}9b)$$

Note that, in the limit of $\mu_k \to 0$, Eqs. 5-9 reduce to Eqs. 5-6 of Sample Problem 5-7.

Let us now find the numerical values of a and T:

$$a = \frac{2.6\ \text{kg} - 9.5\ \text{kg}(\sin 34^\circ - 0.15\cos 34^\circ)}{2.6\ \text{kg} + 9.5\ \text{kg}}(9.80\ \text{m/s}^2)$$

$$= -1.2\ \text{m/s}^2,$$

$$T = \frac{(9.5\ \text{kg})(2.6\ \text{kg})(9.80\ \text{m/s}^2)}{9.5\ \text{kg} + 2.6\ \text{kg}}(1 + \sin 34^\circ - 0.15\cos 34^\circ)$$

$$= 29\ \text{N}.$$

The negative value of a is consistent with the way we set up our equations; the block moves down the plane, as it did in Sample Problem 5-7, but with less acceleration than it did in the frictionless case (2.2 m/s^2).

The tension in the string is less than it was in the frictionless case (31 N). Block 1 accelerates less rapidly down the plane when there is friction, so it doesn't pull as strongly on the string attached to block 2.

One additional question that must be answered is whether the system will move at all. That is, is there enough force down the plane to exceed the static friction and start the motion? When the system is initially at rest, the tension in the string is equal to the weight of m_2, or (2.6 kg) (9.8 m/s^2) = 26 N. The maximum static friction, which opposes the tendency to move down the plane, is $\mu_s N = \mu_s m_1 g \cos\theta = 19$ N. The component of the weight of m_1 acting down the plane is $m_1 g \sin\theta = 52$ N. Thus

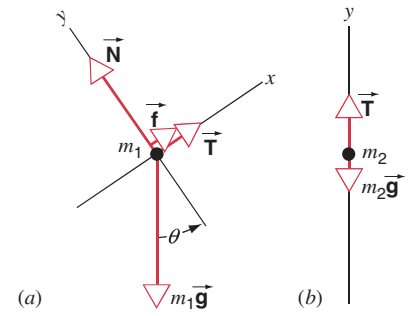


FIGURE 5-17. Sample Problem 5-10. The free-body diagrams of Fig. 5-11, in the case of friction along the plane.

there is more than enough weight acting down the plane (52 N) to overcome the total of the tension and the static frictional force (26 N + 19 N = 45 N), and the system does indeed move. You should be able to show that if the static coefficient of friction is greater than 0.34 then there will be no motion.

5-4 THE DYNAMICS OF UNIFORM CIRCULAR MOTION

As we discussed in Section 4-5, when an object of mass m moves in a circle of radius r at a uniform speed v, it experiences a radial or centripetal acceleration of magnitude v^2/r. In this section we consider the circular motion resulting from several forces acting on the object. Newton's second law must hold in this case in vector form: $\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Since $\vec{\mathbf{a}}$ is always in the radial direction, the net force must also be radial. Its magnitude must be given by

$$\left|\Sigma \vec{\mathbf{F}}\right| = ma = \frac{mv^2}{r}. \qquad (5\text{-}10)$$

Whatever the nature or origin of the forces that act on the object in uniform circular motion, the resultant of all the forces must be (1) in the radial direction, and (2) of magnitude mv^2/r. Even though the magnitude of the object's velocity remains constant, there is an acceleration and thus a net force, because the *direction* of the velocity is changing.

The following examples illustrate applications of Newton's laws to uniform circular motion.

The Conical Pendulum

Figure 5-18 shows a small body of mass m revolving in a horizontal circle with constant speed v at the end of a string of length L. As the body swings around, the string sweeps over the surface of an imaginary cone. This device is called a *conical pendulum*. Let us find the time required for one complete revolution of the body.

If the string makes an angle θ with the vertical, the radius of the circular path is $R = L \sin \theta$. The forces acting on the body of mass m are its weight $m\vec{\mathbf{g}}$ and the tension $\vec{\mathbf{T}}$ of the string, as shown in Fig. 5-18*b*. We can resolve $\vec{\mathbf{T}}$ at any instant into a radial and a vertical component

$$T_r = -T \sin \theta \qquad \text{and} \qquad T_z = T \cos \theta. \qquad (5\text{-}11)$$

The radial component is negative if we define the radial direction to be positive outward from the central axis.

Using the coordinate system shown in Fig. 5-18*b*, we can write the components of the net force on the body as $\Sigma F_r = T_r = -T \sin \theta$ and $\Sigma F_z = T \cos \theta - mg$. Since the body has no vertical acceleration, we can write the z component of Newton's second law as

$$T \cos \theta - mg = 0. \qquad (5\text{-}12)$$

The radial component of Newton's second law is $\Sigma F_r = ma_r$. The radial acceleration is $a_r = -v^2/R$, negative because it acts radially inward and we have chosen the outward radial direction to be positive. For this case Newton's second law gives

$$-T \sin \theta = ma_r = m\left(\frac{-v^2}{R}\right). \qquad (5\text{-}13)$$

Eliminating T between these two equations, we can solve for the speed of the body:

$$v = \sqrt{Rg \tan \theta}. \qquad (5\text{-}14)$$

If we let t represent the time for one complete revolution of the body, then

$$v = \frac{2\pi R}{t}$$

or

$$t = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{Rg \tan \theta}} = 2\pi \sqrt{\frac{R}{g \tan \theta}}.$$

However, $R = L \sin \theta$, so that

$$t = 2\pi \sqrt{\frac{L \cos \theta}{g}}. \qquad (5\text{-}15)$$

This equation gives the relation between t, L, and θ. Note that t, called the *period* of motion, does not depend on m.

If $L = 1.2$ m and $\theta = 25°$, what is the period of the motion? We have

$$t = 2\pi \sqrt{\frac{(1.2 \text{ m})(\cos 25°)}{9.8 \text{ m/s}^2}} = 2.1 \text{ s}.$$

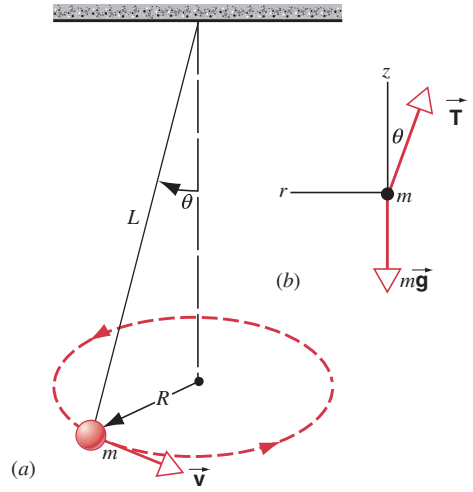


FIGURE 5-18. The conical pendulum. (*a*) A body of mass m suspended from a string of length L moves in a circle; the string describes a right circular cone of semiangle θ. (*b*) A free-body diagram of the body.

The Rotor

In many amusement parks we find a device often called the rotor. The rotor is a hollow cylindrical room that can be set rotating about the central vertical axis of the cylinder. A person enters the rotor, closes the door, and stands up against the wall. The rotor gradually increases its rotational

speed from rest until, at a predetermined speed, the floor below the person is opened downward, revealing a deep pit. The person does not fall but remains "pinned up" against the wall of the rotor. What minimum rotational speed is necessary to prevent falling?

The forces acting on the person are shown in Fig. 5-19. The person's weight is $m\vec{\mathbf{g}}$, the force of static friction between person and rotor wall is $\vec{\mathbf{f}}_s$, and $\vec{\mathbf{N}}$ is the normal force exerted by the wall on the person (which, as we shall see, provides the needed centripetal force). As we did in the previous calculation, we resolve the forces into radial and vertical components, with the positive radial direction outward from the axis of rotation and the positive z axis upward. The components of the net force on the person are then $\Sigma F_r = -N$ and $\Sigma F_z = f_s - mg$. Note that $\vec{\mathbf{N}}$ provides the centripetal force in this case.

The radial acceleration is $a_r = -v^2/R$ and the vertical acceleration is $a_z = 0$. The radial and vertical components of Newton's second law then give

$$-N = ma_r = m\left(\frac{-v^2}{R}\right) \qquad \text{and} \qquad f_s - mg = ma_z = 0.$$

Writing $f_s = \mu_s N$ and substituting $N = mv^2/R$ from the first equation and $f_s = mg$ from the second, we can solve for v to find

$$v = \sqrt{\frac{gR}{\mu_s}}. \tag{5-16}$$

This equation relates the coefficient of friction necessary to prevent slipping to the tangential speed of an object on the wall. Note that the result does not depend on the person's weight.

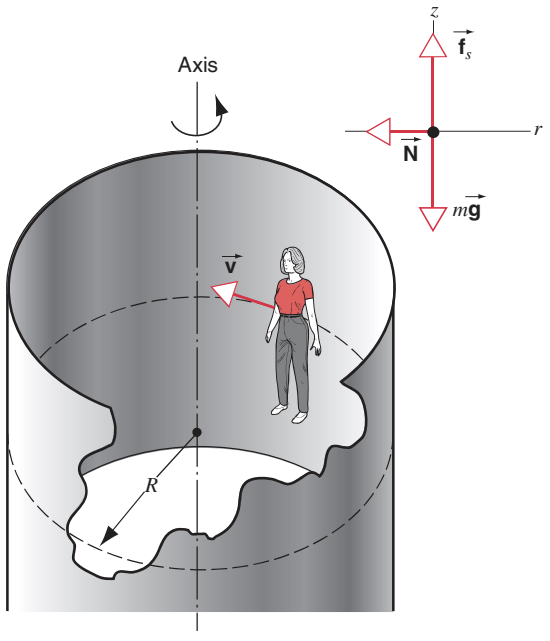


FIGURE 5-19. The rotor. Forces acting on the person are shown.

As a practical matter the coefficient of friction between the textile material of clothing and a typical rotor wall (canvas) is about 0.40. For a typical rotor the radius is 2.0 m, so that v must be about 7.0 m/s or more. The circumference of the circular path is $2\pi R = 12.6$ m, and at 7.0 m/s it takes a time of $t = 12.6$ m/(7.0 m/s) = 1.80 s to complete each revolution. The rotor must therefore turn at a rate of at least 1 revolution/1.80 s = 0.56 revolution/s or about 33 rpm, the same rate of rotation as a phonograph turntable.

The Banked Curve

Let the block in Fig. 5-20*a* represent an automobile or railway car moving at constant speed v on a *level* roadbed around a curve having a radius of curvature R. In addition to two vertical forces—namely, the weight $m\vec{\mathbf{g}}$ and a normal force $\vec{\mathbf{N}}$—a horizontal force $\vec{\mathbf{P}}$ must act on the car. The force $\vec{\mathbf{P}}$ provides the centripetal force necessary for motion in a circle. In the case of the automobile this force is supplied by a sidewise frictional force exerted by the road on the tires; in the case of the railway car the force is supplied by the rails exerting a sidewise force on the inner rims of the car's wheels. Neither of these sidewise forces can safely be relied on to be large enough at all times, and both cause

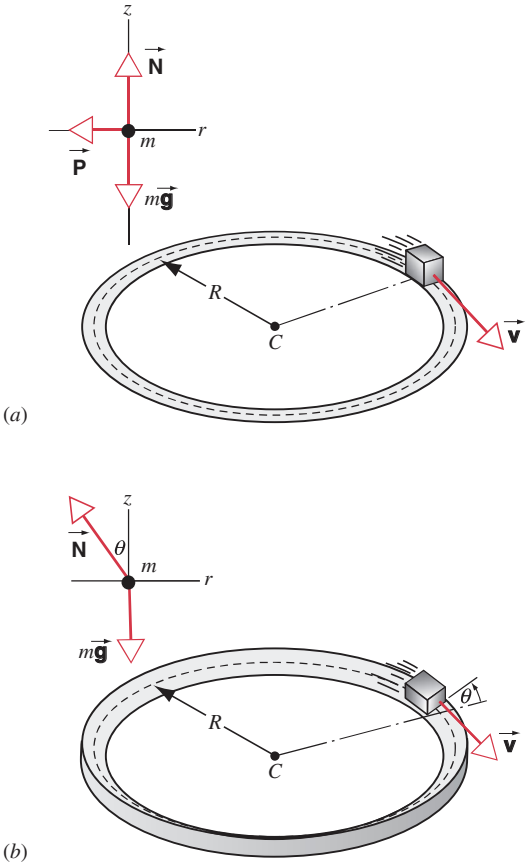


FIGURE 5-20. (*a*) A level roadbed. A free-body diagram of the moving body is shown at left. The centripetal force must be supplied by friction between tires and road. (*b*) A banked roadbed. No friction is necessary to round the curve safely.

unnecessary wear. Hence, the roadbed is *banked* on curves, as shown in Fig. 5-20*b*. In this case, the normal force $\vec{\mathbf{N}}$ has not only a vertical component, as before, but also a horizontal component that supplies the centripetal force necessary for uniform circular motion. Thus no additional sidewise forces are needed with a roadbed that is properly banked for vehicles of a particular speed.

The correct angle θ of banking in the absence of friction can be obtained as follows. We begin, as usual, with Newton's second law, and we refer to the free-body diagram shown in Fig. 5-20*b*. The radial and vertical components of the net force on the moving body are $\Sigma F_r = -N \sin\theta$ and $\Sigma F_z = N\cos\theta - mg$. As before, the radial acceleration is $a_r = -v^2/R$ and the vertical acceleration is $a_z = 0$, so we can write the components of Newton's second law as

$$-N\sin\theta = ma_r = m\left(\frac{-v^2}{R}\right)$$

and

$$N\cos\theta - mg = ma_z = 0.$$

Solving these two equations for $\sin\theta$ and $\cos\theta$ and dividing the resulting expressions, we obtain

$$\tan\theta = \frac{v^2}{Rg}. \tag{5-17}$$

Note that the proper angle of banking depends on the speed of the car and the curvature of the road. It does not depend on the mass of the car; for a given banking angle, all cars will be able to travel safely. For a given curvature, the road is banked at an angle corresponding to an expected average speed. Curves are often marked by signs giving the proper speed for which the road was banked. If vehicles exceed that speed, the friction between tires and road must supply the additional centripetal force needed to travel the curve safely.

Check the banking formula for the limiting cases $v = 0$, $R \longrightarrow \infty$, v large, and R small. Also, note that Eq. 5-17, if solved for v, gives the same result that we derived for the speed of the bob of a conical pendulum. Compare Figs. 5-18*b* and 5-20*b*, noting their similarities.

5-5 TIME-DEPENDENT FORCES (Optional)*

In Chapter 2, we analyzed the braking of an automobile by assuming the acceleration to be constant. In practice, this is seldom the case. Under many circumstances, especially at high speed, we usually apply the brakes slowly at first and then more strongly as the car slows. The braking force therefore depends on the time during the interval over which the car is slowing, and so the acceleration $a(t)$ is a function of how we apply the brakes.

Even though the force is not constant, we can still use Newton's laws to analyze the motion, but we cannot use the equations of Chapter 2 to find the position and velocity as functions of the time, because those equations were derived for constant acceleration. We assume here for simplicity that the forces and the motion are in one dimension, which we take to be the x direction. We can find the x component of the net force $F_x(t)$ in the usual way from a free-body diagram, and then we continue by writing $a_x = dv_x/dt$ and using Newton's second law:

$$a_x(t) = \frac{dv_x}{dt} = \frac{F_x(t)}{m}$$

or

$$dv_x = \frac{F_x(t)}{m}\,dt. \tag{5-18}$$

Suppose the object begins its motion at $t = 0$ with initial velocity v_{0x}. What is its velocity v_x at time t? We integrate Eq. 5-18 on the left between v_{0x} and v_x, and on the right between 0 and t:

$$\int_{v_{0x}}^{v_x} dv_x = \int_0^t \frac{F_x(t)}{m}\,dt.$$

$$v_x - v_{0x} = \frac{1}{m}\int_0^t F_x(t)\,dt$$

so

$$v_x(t) = v_{0x} + \frac{1}{m}\int_0^t F_x(t)\,dt. \tag{5-19}$$

Note that this reduces to Eq. 2-26 if F_x is a constant ($= ma_x$) and so can be taken out of the integral.

Continuing in the same way with $v_x = dx/dt$, we can find the position as a function of the time

$$x(t) = x_0 + \int_0^t v_x(t)\,dt. \tag{5-20}$$

This reduces to Eq. 2-28 when F_x is a constant, in which case $v_x(t) = v_{0x} + a_x t$.

When we have a force that depends on the time, we can use Eqs. 5-19 and 5-20 to find analytical expressions for $v_x(t)$ and $x(t)$. We have already seen in Section 4-4 how this can be done in a similar way for a force that depends on the velocity. More often, especially when there is no analytical expression for the integrals, we find it is necessary or convenient to use numerical or computational methods.

SAMPLE PROBLEM 5-11. A car of mass $m = 1260$ kg is moving at 105 km/h (about 65 mi/h or 29.2 m/s). The driver begins to apply the brakes so that the magnitude of the braking force increases linearly with time at the rate of 3360 N/s. (*a*) How much time passes before the car comes to rest? (*b*) How far does the car travel in the process?

*This section involves integral calculus and can be postponed until the student is more familiar with integration methods.

Solution (*a*) If we choose the direction of the car's velocity as the positive x direction, then we can represent the braking force as $F_x = -ct$ where $c = 3360$ N/s. (The negative sign indicates that the direction of the braking force is opposite that of the velocity). Using Eq. 5-19, we then have

$$v_x(t) = v_{0x} + \frac{1}{m}\int_0^t (-ct)\,dt = v_{0x} - \frac{ct^2}{2m}.$$

To find the time t_1 when the car comes to rest, we set this expression for $v_x(t)$ equal to zero and solve for t:

$$t_1 = \sqrt{\frac{2v_{0x}m}{c}} = \sqrt{\frac{2(29.2\text{ m/s})(1260\text{ kg})}{3360\text{ N/s}}} = 4.68\text{ s}.$$

(*b*) To find how far the car travels in this time, we need an expression for $x(t)$, for which we must integrate $v_x(t)$ according to Eq. 5-20:

$$x(t) = x_0 + \int_0^t \left(v_{0x} - \frac{ct^2}{2m}\right)dt = x_0 + v_{0x}t - \frac{ct^3}{6m}.$$

Evaluating this expression at $t = t_1$ (setting x_0 to 0), we obtain

$$x(t_1) = 0 + (29.2\text{ m/s})(4.68\text{ s}) - \frac{(3360\text{ N/s})(4.68\text{ s})^3}{6(1260\text{ kg})} = 91.1\text{ m}.$$

Figure 5-21 shows the time dependence of x, v_x, and a_x. In contrast with the case of constant acceleration, $v_x(t)$ is not a straight line.

With this method of braking, most of the change in velocity occurs near the end of the motion. The change in velocity in the first second after the brakes are applied is only 1.3 m/s (about 3 mi/h); in the last second, however, the change is 11.2 m/s (about 25 mi/h). (Recall that in the case of constant acceleration, the change in velocity is the same in equal time intervals.) Can you think of an advantage to braking in this manner? Are there also disadvantages?

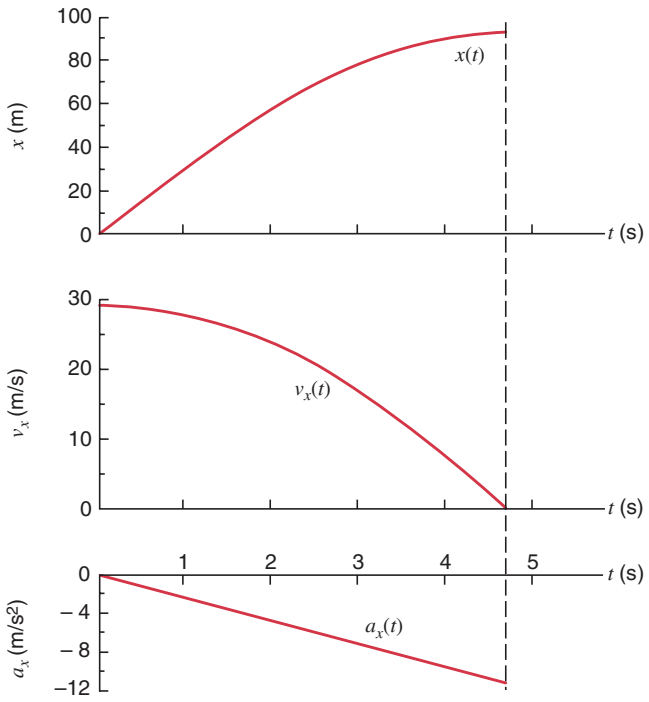


FIGURE 5-21. Sample Problem 5-11. The deduced position $x(t)$ and velocity $v_x(t)$ are shown corresponding to $a_x(t)$, which varies linearly with time. The dashed line marks the instant ($t = 4.68$ s) at which the car comes to rest.

5-6 NONINERTIAL FRAMES AND PSEUDOFORCES (Optional)

In our treatment of classical mechanics thus far, we have assumed that measurements and observations were made from an inertial reference frame. This is one of the set of reference frames defined by Newton's first law—namely, that set of frames in which a body will not be accelerated ($\vec{\mathbf{a}} = 0$) if there are no force-producing bodies in its environment ($\Sigma\,\vec{\mathbf{F}} = 0$). The choice of a reference frame is always ours to make, so that if we choose to select only inertial frames, we do not restrict in any way our ability to apply classical mechanics to natural phenomena.

Nevertheless, if we find it convenient, we can apply classical mechanics from the point of view of an observer in a *noninertial frame*—that is, a frame attached to a body that is accelerating as viewed from an inertial frame. The frames of an accelerating car or a rotating merry-go-round are examples of noninertial frames.

To apply classical mechanics in noninertial frames we must introduce additional forces known as *pseudoforces* (sometimes called inertial forces). Unlike the forces we have examined thus far, we cannot associate pseudoforces with any particular object in the environment of the body on which they act, and we cannot classify them into any of the categories discussed in Section 5-1. Moreover, if we view the body from an inertial frame, the pseudoforces disappear. Pseudoforces are simply devices that permit us to apply classical mechanics in the normal way to events if we insist on viewing the events from a noninertial reference frame.

As an example, consider an observer S' riding in a van that is moving at constant velocity. The van contains a long airtrack with a frictionless 0.25-kg glider resting at one end (Fig. 5-22*a*). The driver of the van applies the brakes, and the van begins to decelerate. An observer S on the ground measures the constant acceleration of the van to be -2.8 m/s^2. The observer S' riding in the van is therefore in a noninertial frame of reference when the van begins to decelerate. S' observes the glider to move down the track with an acceleration of $+2.8$ m/s^2 (Fig. 5-22*b*). How might each observer use Newton's second law to account for the motion of the glider?

For ground observer S, who is in an inertial reference frame, the analysis is straightforward. The glider, which had been moving forward at constant velocity before the van started to brake, simply continues to do so. According to S, the glider has no acceleration and therefore no horizontal force need be acting on it.

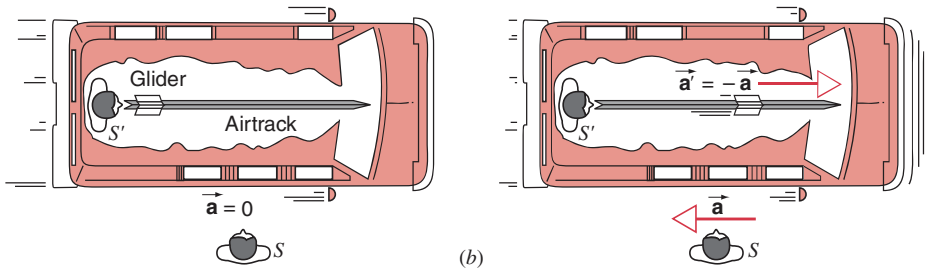


FIGURE 5-22. (*a*) Ground-based observer S watches observer S' traveling in a van at constant velocity. The van is traveling to the right, which we take to be the positive x direction. Both observers are in inertial reference frames. (*b*) The van brakes with constant acceleration a according to observer S. Observer S', now in a noninertial reference frame, sees the glider move forward on its airtrack with constant acceleration $\vec{\mathbf{a}}' = -\vec{\mathbf{a}}$. Observer S' accounts for this motion in terms of a pseudoforce.

Observer S', however, sees the glider accelerate and can find no object in the environment of the glider that exerted a force on it to provide its observed forward acceleration. To preserve the applicability of Newton's second law, S' must assume that a force (in this case a pseudoforce) acts on the glider. According to S', this force $\vec{\mathbf{F}}'$ must equal $m\vec{\mathbf{a}}'$, where $\vec{\mathbf{a}}'$ $(=-\vec{\mathbf{a}})$ is the acceleration of the glider measured by S'. The x component of this pseudoforce is

$$F'_x = ma'_x = (0.25\text{ kg})(2.8\text{ m/s}^2) = 0.70\text{ N},$$

and its direction is the same as $\vec{\mathbf{a}}'$—that is, toward the front of the van. This force, which is very real from the point of view of S', is not apparent to ground-based observer S, who has no need to introduce it to account for the motion of the glider.

One indication that pseudoforces are non-Newtonian is that they violate Newton's third law. To apply Newton's third law, S' must find a reaction force exerted *by* the glider *on* some other body. No such reaction force can be found, and so Newton's third law is violated.

Pseudoforces are very real to those that experience them. Imagine yourself riding in a car that is rounding a curve to the left. To a ground observer, the car is experiencing a centripetal acceleration and therefore constitutes a noninertial reference frame. If the car has smooth vinyl seats, you will find yourself sliding across the seat to the right. To the ground observer, who is in an inertial frame, this is quite natural; your body is simply trying to obey Newton's first law and move in a straight line, and it is the car that is sliding to the left under you. From your point of view in the noninertial reference frame of the car, you must ascribe your sliding motion to a pseudoforce pulling you to the right. This type of pseudoforce is called a *centrifugal force,* meaning a force directed *away from* the center.

Riding on a merry-go-round, you are again in an accelerated and therefore noninertial reference frame in which objects will apparently move outward from the axis of rotation under the influence of the centrifugal force. If you hold a ball in your hand, it seems to you that the net horizontal force on the ball is zero, the outward centrifugal force being balanced by the inward force exerted on the ball by your hand. To a ground observer, who is in an inertial reference frame, the ball is moving in a circle, accelerating toward the center under the influence of the *centripetal* force you exert on it with your hand. To the ground observer, there is no centrifugal force because the net force on the ball is *not* zero: it is accelerating radially inward.

Pseudoforces can be used as the basis of practical devices. Consider the centrifuge, one of the most useful of laboratory instruments. As a mixture of substances moves rapidly in a circle, the more massive substances experience a larger centrifugal force mv^2/r and move farther away from the axis of rotation. The centrifuge thus uses a pseudoforce to separate substances by mass, just as the mass spectrometer (Section 3-4) uses an electromagnetic force to separate atoms by mass.

Another pseudoforce is called a *Coriolis* force. Suppose that you roll a ball inward with constant speed along a radial line painted on the floor of a rotating merry-go-round. At the instant you release it at the radius r, it has just the right tangential velocity (the same as yours) to be in circular motion. As it moves inward it would take a smaller tangential speed to maintain its circular motion at the same rate as its immediate surroundings. Because it has no way to lose tangential speed (we assume little friction between the ball and the floor), it moves a bit ahead of the painted line representing a uniform rotational speed. That is, in your rotating noninertial reference frame you would suggest that a sideways pseudoforce—a Coriolis force—causes the ball to veer steadily away from the line as it rolls inward. To a ground observer in an inertial frame, there is no Coriolis force: the ball moves in a straight line at a speed determined by the components of its velocity at the instant of release.

Perhaps the most familiar example of the effects of the Coriolis force is in the motion of the atmosphere around centers of low or high pressure. Figure 5-23 shows a diagram of a low-pressure center in the northern hemisphere.

Because the pressure is lower than the surroundings, air flows radially inward in all directions. As the Earth rotates (making it a noninertial frame), the effect is similar to that of the ball on the merry-go-round: air rushing inward from the south moves a bit ahead of an imaginary line drawn on the rotating Earth, while air from the north (like a ball rolled outward on the merry-go-round) lags a bit behind the line. The total effect is that the air rotates in a counterclockwise direction around the low-pressure center. This Coriolis effect is thus responsible for the circulation of winds in a cyclone or hurricane. In the southern hemisphere the effects are reversed.

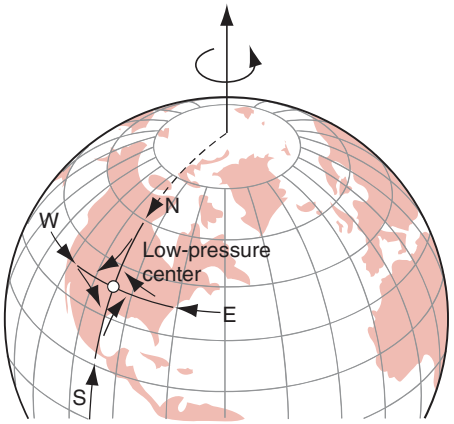
It is necessary to correct for the Coriolis effect of the rotating Earth in the motion of long-range artillery shells. For a typical shell of range 10 km, the Coriolis effect may cause a deflection as large as 20 m. Such corrections are built into the computer programs used to control the aiming and firing of long-range weapons. Things can go wrong, however, as the British Navy discovered in a World War I battle near the Falkland Islands. Its fire control manuals were written for the northern hemisphere, and the Falklands are in the southern hemisphere where the Coriolis correction would be in the opposite direction. The British shells were landing about 100 m from their targets, because the correction for the Coriolis effect was being made in the wrong direction!

In mechanical problems, then, we have two choices: (1) select an *inertial* reference frame and consider only "real" forces—that is, forces that we can associate with definite bodies in the environment; or (2) select a *noninertial* reference frame and consider not only the "real" forces but suitably defined pseudoforces. Although we usually choose the first alternative, we sometimes choose the second; both are completely equivalent and the choice is a matter of convenience. ■

5-7 LIMITATIONS OF NEWTON'S LAWS (Optional)

In the first five chapters, we have described a system for analyzing mechanical behavior with a seemingly vast range of applications. With little more than the equations of Newton's laws, you can design great skyscrapers and suspension bridges, or even plan the trajectory of an interplanetary spacecraft (Fig. 5-24). Newtonian mechanics, which provided these computational tools, was the first truly revolutionary development in theoretical physics.

Here is an example of our faith in Newton's laws. Galaxies and clusters of galaxies are often observed to rotate, and by observation we can deduce the speed of rotation. From this we can calculate the amount of matter that must be present in the galaxy or cluster for gravity to supply the centripetal force corresponding to the observed rotation. Yet the amount of matter that we actually observe with telescopes is far less than we expect. Therefore, it has been proposed that there is additional "dark matter" that we can-



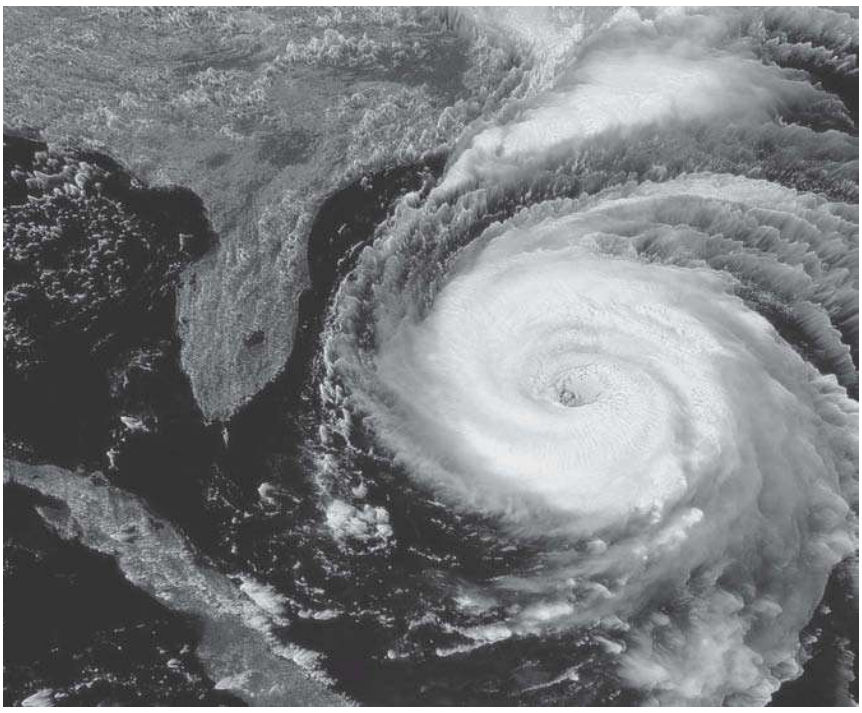


FIGURE 5-23. A low-pressure center on the rotating Earth. As the air flows inward, it appears to rotate counterclockwise to noninertial observers in the northern hemisphere of the rotating Earth. A hurricane (photo) is such a low-pressure center.

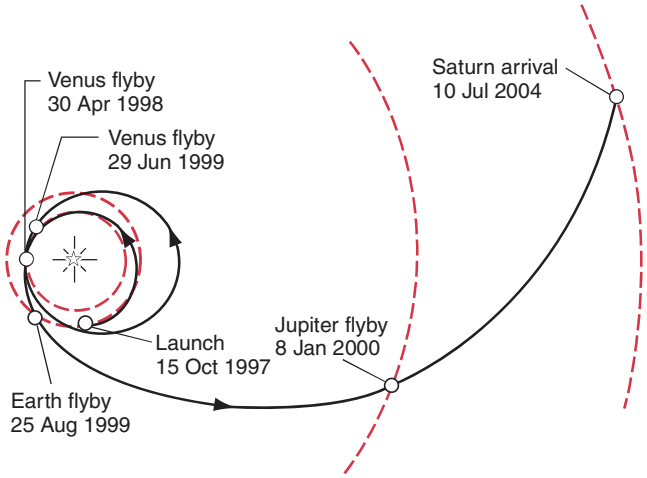


FIGURE 5-24. The trajectory of the Cassini mission to Saturn, launched from Earth on October 15, 1997. The ability to calculate such trajectories with pinpoint accuracy is a triumph of the methods of classical mechanics. The four planetary flybys are used to provide "gravity assists" that increase the speed of the spacecraft (see Section 6-1) and enable it to reach Saturn. To learn more about this mission, see the Web site at http://www.jpl.nasa.gov/cassini.

not see with telescopes but that must be present to provide the needed gravitational force. There is as yet no convincing candidate for the type or nature of this dark matter, and so other explanations have been proposed for the apparent inconsistency between the amount of matter actually observed in the galaxies and the amount we think is needed to satisfy Newton's laws. One proposed explanation is that our calculations are incorrect because Newton's laws do not hold in the conditions that we find on the very large scale—that is, when the accelerations are very small (below a few times 10^{-10} m/s^2). In particular, it has been proposed that for these very small accelerations, the force is proportional to a^2 instead of a.

Figure 5-25 shows the results of an experiment testing this supposition. If force depended on the acceleration to some power other than 1, the data would not fall on a straight line. From this extremely precise experiment, we conclude that down to accelerations of about 10^{-10} m/s^2, force is proportional to acceleration and Newton's second law holds.

In the 20th century, we have experienced three other revolutionary developments: Einstein's special theory of relativity (1905), his general theory of relativity (1915), and quantum mechanics (in about 1925). Special relativity teaches us that we cannot extrapolate the use of Newton's laws to particles moving at speeds comparable to the speed of light. General relativity shows that we cannot use Newton's laws in the vicinity of extremely massive objects. Quantum mechanics teaches us that we cannot extrapolate Newton's laws to objects as small as atoms.

Special relativity, which involves a distinctly non-Newtonian view of space and time, can be applied under all circumstances, at both high speeds and low speeds. In the limit of low speeds, it can be shown that the dynamics of special relativity reduces directly to Newton's laws. Similarly, general relativity can be applied to weak as well as strong gravitational forces, but its equations reduce to Newton's laws for weak forces. Quantum mechanics can be applied to individual atoms, where a certain randomness of behavior is predicted, or to ordinary objects containing a huge number of atoms, in which case the randomness averages out to give Newton's laws once again.

Within the past two decades, another apparently revolutionary development has emerged. This new development concerns mechanical systems whose behavior is described as *chaotic*. One of the hallmarks of Newton's laws is their ability to predict the future behavior of a system, if we know the forces that act and the initial motion. For example, from the initial position and velocity of a space probe that experiences known gravitational forces from the Sun and the planets, we can calculate its exact trajectory. On the other hand, consider a twig floating in a turbulent stream. Even though it is acted on at all times by forces governed by Newtonian mechanics, its path downstream is totally unpredictable. If two twigs are released side-by-side in the stream, they may be found very far apart downstream. One particular theme of chaotic dynamics is that tiny changes in the initial conditions of a problem can be greatly amplified and can cause substantial differences in the predicted outcomes. Chaotic dynamics is often invoked in weather forecasting, and it has been said that the fluttering of a butterfly over Japan could be related to the subsequent development of a hurricane over the Gulf of Mexico.

Such chaotic motions occur not only in complex systems like a turbulent stream but also in simple physical systems as well, such as a pendulum, a slowly dripping faucet, or an oscillating electrical circuit. In the 1960s it was discovered that the seemingly chaotic behavior of these sys-

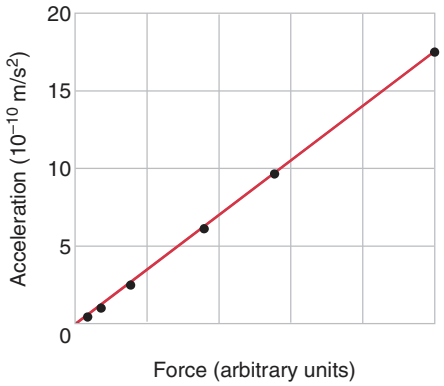


FIGURE 5-25. Results of an experiment to test whether Newton's second law holds for small accelerations below 10^{-9} m/s^2. The straight line shows that acceleration is proportional to the applied force down to 10^{-10} m/s^2, and so Newton's law remains valid even at such small accelerations.

tems conceals a hidden order and regularity, the study of which has formed the core of a new branch of science, *chaos.** Applications of the laws of chaos have been found not only in physical systems but in biological systems as well. Even areas of social science, such as economics and population dynamics, show chaotic behavior.

Calculations combining the Newtonian mechanics of particles with chaos theory have shown that the orbit of the planet Pluto is chaotic on a time scale of tens of millions of years (a short time compared with the age of the solar system, about 4.5 billion years, but a long time compared with Pluto's orbital period around the Sun, about 250 years). Chaos theory has also been used to explain two properties of the asteroid belt (which lies between the orbits of Mars and Jupiter) that could not be understood within the framework of conventional Newtonian mechanics: (1) many asteroids stray from what should be stable orbits, some of them becoming meteorites that rain steadily down on Earth, and (2) within the asteroid belt are several empty gaps where the number of orbiting asteroids is small or zero. It has been only within the past decade that high-speed computers have permitted detailed calculations of the dynamics of such systems to be followed for the time scales necessary to observe this unusual behavior, and as additional calculations are done, new applications of this exciting field continue to be discovered. ■

*See *Chaos—Making a New Science,* by James Gleick (Penguin Books, 1987).

MULTIPLE CHOICE

5-1 Force Laws

5-2 Tension and Normal Forces

1. A spring balance is attached on both ends to strings; the strings hang over frictionless pulleys and are each connected to 20 N weights as shown in Fig. 5-26. The reading on the scale will be closest to

(A) 0 N (B) 10 N (C) 20 N (D) 40 N

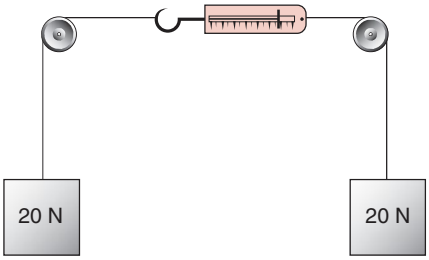


FIGURE 5-26. Multiple-choice question 1.

2. Which of the following statements is most correct?

(A) The normal force is the same thing as the weight.
(B) The normal force is different from the weight, but always has the same magnitude.
(C) The normal force is different from the weight, but the two form an action–reaction pair according to Newton's third law.
(D) The normal force is different from the weight, but the two may have the same magnitude in certain cases.

3. A wooden box is sitting on a table. The normal force on the box from the table is 75 N. A second identical box is placed on top of the first box. The normal force on the first box by the table will

(A) decrease. (B) remain at 75 N.
(C) increase to 150 N. (D) increase to 300 N.

4. A woman can stand in running shoes or stiletto high heels on a level surface. Assuming her total mass is the same regardless of the shoes she wears, then the normal force on her shoes by the ground is

(A) greater in the case of the running shoes, because of the larger area of contact with the ground.
(B) the same for either pair of shoes.
(C) greater for the stiletto heels, because of the smaller area of contact with the ground.
(D) solely dependent on whether she is standing with her knees bent.

5. A *real* rope is hanging by one end from the ceiling. The other end dangles freely. If the mass of the rope is 100 g, then the tension is

(A) 0.98 N along the entire length of the rope.
(B) 0.49 N along the entire length of the rope.
(C) 0.98 N at the bottom of the rope, and varies linearly to zero at the top of the rope.
(D) 0.98 N at the top of the rope, and varies linearly to zero at the bottom of the rope.

6. A bird of weight W is resting at the center of a stretched wire of negligible mass. Each half of the wire makes a small angle with the horizontal. What can be concluded about the tension T in the wire?

(A) $T < W/2$ (B) $W/2 \leq T \leq W$ (C) $T > W$
(D) More information is needed to answer the question.

5-3 Frictional Forces

7. Which statement is correct about the weight of an object and the force of kinetic friction on that object?

(A) The weight is always greater than the frictional force.
(B) The weight is always equal to the frictional force.
(C) The weight is less than the frictional force for sufficiently light objects.
(D) The weight can be more or less than the frictional force.

8. A 2.0-kg block of wood is on a level surface where $\mu_s = 0.80$ and $\mu_k = 0.60$. A 13.7-N force is being applied to the block parallel to the surface.

(*a*) If the block was originally at rest, then

(A) it will remain at rest, and the force of friction will be about 15.7 N.
(B) it will remain at rest, and the force of friction will be about 13.7 N.
(C) it will remain at rest, and the force of friction will be 11.8 N.
(D) it will begin to slide with a net force of about 1.9 N acting on the block.

(*b*) If the block was originally in motion, and the 13.7-N applied force is in the direction of motion, then

(A) it will accelerate under a net force of about 1.9 N.
(B) it will move at constant speed.
(C) it will decelerate under a net force of about 1.9 N.
(D) it will decelerate under a net force of 11.8 N.

9. Two similar wooden blocks are tied one behind the other and pulled across a level surface. Friction is not negligible. The force required to pull them at constant speed is F. If one block is stacked upon the other then the new force required to pull them at constant speed will be approximately

(A) $F/2$. (B) F. (C) $\sqrt{2}F$. (D) $2F$.

10. Automatic braking systems (ABS) on automobiles prevent the tires from locking by sensing when the tires stop spinning and then reducing the braking force until they begin to spin again. Knowing that $\mu_s > \mu_k > \mu_{rolling}$, an automobile equipped with ABS will

(A) always stop in a shorter distance
(B) stop in shorter distance on dry pavement but not on wet pavement
(C) stop in about the same distance
(D) always stop in a longer distance

than an automobile that stops by locking the tires.

11. A 1.0-kg block of wood sits on top of an identical block of wood, which sits on top of a flat level table made of plastic. The coefficient of static friction between the wood surfaces is μ_1, and the coefficient of static friction between the wood and plastic is μ_2.

(*a*) A horizontal force F is applied to the top block only, and this force is increased until the top block starts to slide. The bottom block will slide with the top block if and only if

(A) $\mu_1 < \frac{1}{2}\mu_2$. (B) $\frac{1}{2}\mu_2 < \mu_1 < \mu_2$.
(C) $\mu_2 < \mu_1$. (D) $2\mu_2 < \mu_1$.

(*b*) Instead a horizontal force F is applied to the bottom block only, and this force is increased until the bottom block just starts to slide. Under what conditions will the top block slide with the bottom block?

(A) If $\mu_1 > 0$ the top block will slide regardless of μ_2.
(B) $\frac{1}{2}\mu_2 < \mu_1 < \mu_2$ (C) $\mu_2 < \mu_1$ (D) $2\mu_2 < \mu_1$

5-4 The Dynamics of Uniform Circular Motion

12. A motorcycle moves around a vertical circle with a constant speed under the influence of the force of gravity $\vec{\mathbf{W}}$, the force of friction between the wheels and the track $\vec{\mathbf{f}}$, and the normal force between the wheels and the track $\vec{\mathbf{N}}$.

(*a*) Which of the following quantities has a constant magnitude?

(A) $\vec{\mathbf{N}}$ (B) $\vec{\mathbf{N}} + \vec{\mathbf{f}}$ (C) $\vec{\mathbf{f}} + \vec{\mathbf{W}}$ (D) $\vec{\mathbf{N}} + \vec{\mathbf{W}} + \vec{\mathbf{f}}$

(*b*) Which of the following quantities, when nonzero, is always directed toward the center of the circle?

(A) $\vec{\mathbf{f}}$ (B) $\vec{\mathbf{W}}$ (C) $\vec{\mathbf{f}} + \vec{\mathbf{W}}$ (D) $\vec{\mathbf{N}} + \vec{\mathbf{f}}$

13. An automobile drives over some hilly terrain. The motion of the automobile at the top of a hill is instantaneously similar to circular motion with the center of curvature beneath the road. The motion of the automobile at the bottom of a dip is instantaneously similar to circular motion with the center of curvature above the road. At any time there are three forces on the car: weight $\vec{\mathbf{W}}$, normal $\vec{\mathbf{N}}$, and friction $\vec{\mathbf{f}}$ of the tires with the road. The magnitudes of these three forces are given respectively by W, N, and f.

(*a*) When the car is at the top of a hill, which of the following gives the magnitude of the centripetal force?

(A) N (B) $W + N$ (C) $W - N$ (D) $N - W$

(*b*) When the car is at the bottom of a dip, which of the following gives the magnitude of the centripetal force?

(A) N (B) $W + N$ (C) $W - N$ (D) $N - W$

5-5 Time-Dependent Forces

5-6 Noninertial Frames and Pseudoforces

5-7 Limitations of Newton's Laws

QUESTIONS

1. You can pull a wagon with a rope, but you can't push it with a rope. Is there such a thing as a "negative" tension?

2. There is a limit beyond which further polishing of a surface increases rather than decreases frictional resistance. Explain why.

3. A crate, heavier than you are, rests on a rough floor. The coefficient of static friction between the crate and the floor is the same as that between the soles of your shoes and the floor. Can you push the crate across the floor? See Fig. 5-27.

4. In baseball, a base runner can usually get to a base quicker by running than by sliding. Explain why this is so. Why slide then?

5. How could a person who is at rest on completely frictionless ice covering a pond reach shore? Could she do this by walking, rolling, swinging her arms, or kicking her feet? How could a person be placed in such a position in the first place?

6. Why do tires grip the road better on level ground than they do when going uphill or downhill?

FIGURE 5-27. Question 3.

7. What is the purpose of curved surfaces, called spoilers, placed on the rear of sports cars? They are designed so that air flowing past exerts a downward force.
8. Two surfaces are in contact but are at rest relative to each other. Nevertheless, each exerts a force of friction on the other. Explain.
9. Your car skids across the centerline on an icy highway. Should you turn the front wheels in the direction of the skid or in the opposite direction (*a*) when you want to avoid a collision with an oncoming car and (*b*) when no other car is near but you want to regain control of the steering? Assume rear-wheel drive, then front-wheel drive.
10. Why is it that racing drivers actually speed up when traversing a curve?
11. You are flying a plane at constant altitude and you wish to make a 90° turn. Why do you bank in order to do so?
12. When a wet dog shakes itself, people standing nearby tend to get wet. Why does the water fly outward from the dog in this way?
13. You must have noticed (Einstein did) that when you stir a cup of tea, the floating tea leaves collect at the center of the cup rather than at the outer rim. Can you explain this? (Einstein could.)
14. Suppose that you need to measure whether a tabletop in a train is truly horizontal. If you use a spirit level, can you determine this when the train is moving down or up a grade? When the train is moving along a curve? (Hint: There are two horizontal components.)
15. In the conical pendulum, what happens to the period and the speed when $\theta = 90°$? Why is this angle not achievable physically? Discuss the case for $\theta = 0°$.
16. A coin is put on a phonograph turntable. The motor is started but, before the final speed of rotation is reached, the coin flies off. Explain why.
17. A car is riding on a country road that resembles a roller coaster track. If the car travels with uniform speed, compare the force it exerts on a horizontal section of the road to the force it exerts on the road at the top of a hill and at the bottom of a hill. Explain.
18. You are driving a station wagon at uniform speed along a straight highway. A beach-ball rests at the center of the wagon bed and a helium-filled balloon floats above it, touching the roof of the wagon. What happens to each if you (*a*) turn a corner at constant speed or (*b*) apply the brakes?
19. How does the Earth's rotation affect the measurement of the weight of an object at the equator?
20. Explain why a plumb bob will not hang exactly in the direction of the Earth's gravitational attraction (toward the center of the Earth) at most latitudes.
21. Astronauts in the orbiting space shuttle want to keep a daily record of their weight. Can you think how they might do it, considering that they are "weightless"?
22. Explain how the question "What is the linear velocity of a point on the equator?" requires an assumption about the reference frame used. Show how the answer changes as you change reference frames.
23. What is the distinction between inertial reference frames and those differing only by a translation or rotation of the axes?
24. A passenger in the front seat of a car finds himself sliding toward the door as the driver makes a sudden left turn. Describe the forces on the passenger and on the car at this instant if the motion is viewed from a reference frame (*a*) attached to the Earth and (*b*) attached to the car.
25. Do you have to be concerned with the Coriolis effect when playing tennis or golf? If not, why not?
26. Suppose that you are standing on the balcony of a tall tower, facing east. You drop an object so that it falls to the ground below; see Fig. 5-28. Suppose also that you can locate the impact point very precisely. Will the object strike the ground at *a*, vertically below the release point, at *b* to the east, or at *c* to the west? The object was released from rest; the Earth rotates from west to east.

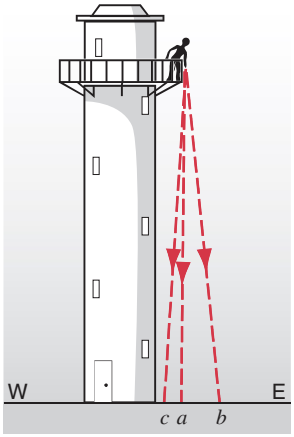


FIGURE 5-28. Question 26.

27. Show by a qualitative argument that, because of the rotation of the Earth, a wind in the northern hemisphere blowing from north to south will be deflected to the right. What about a wind that is blowing from south to north? What is the situation in the southern hemisphere?

EXERCISES

5-1 Force Laws

5-2 Tension and Normal Forces

1. A charged sphere of mass 2.8×10^{-4} kg is suspended from a string. An electric force acts horizontally on the sphere so that the string makes an angle of 33° with the vertical when at rest. Find (*a*) the magnitude of the electric force and (*b*) the tension in the string.
2. An elevator weighing 6200 lb is pulled upward by a cable with an acceleration of 3.8 ft/s^2. (*a*) What is the tension in the cable? (*b*) What is the tension when the elevator is accelerating downward at 3.8 ft/s^2 but is still moving upward?
3. A lamp hangs vertically from a cord in a descending elevator. The elevator has a deceleration of 2.4 m/s^2 before coming to a stop. (*a*) If the tension in the cord is 89 N, what is the mass of

the lamp? (*b*) What is the tension in the cord when the elevator ascends with an upward acceleration of 2.4 m/s^2?

4. An elevator and its load have a combined mass of 1600 kg. Find the tension in the supporting cable when the elevator, originally moving downward at 12.0 m/s, is brought to rest with constant acceleration in a distance of 42.0 m.

5. A 110-kg man lowers himself to the ground from a height of 12 m by holding on to a rope passed over a frictionless pulley and attached to a 74-kg sandbag. (*a*) With what speed does the man hit the ground? (*b*) Is there anything he could do to reduce the speed with which he hits the ground?

6. An 11-kg monkey is climbing a massless rope attached to a 15-kg log over a frictionless tree limb. (*a*) With what minimum acceleration must the monkey climb up the rope so that it can raise the 15-kg log off the ground? If, after the log has been raised off the ground, the monkey stops climbing and hangs on to the rope, what will now be (*b*) the monkey's acceleration and (*c*) the tension in the rope?

7. Figure 5-29 shows a section of an alpine cable-car system. The maximum permitted mass of each car with occupants is 2800 kg. The cars, riding on a support cable, are pulled by a second cable attached to each pylon. What is the difference in tension between adjacent sections of pull cable if the cars are accelerated up to 35° incline at 0.81 m/s^2?

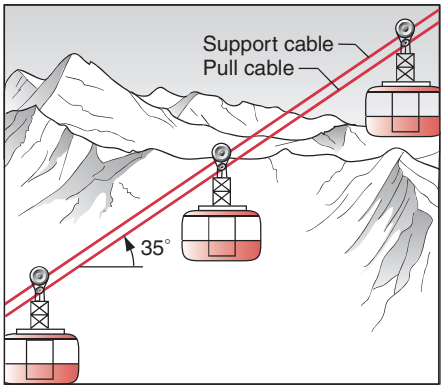


FIGURE 5-29. Exercise 7.

8. The man in Fig. 5-30 weighs 180 lb; the platform and attached frictionless pulley weigh a total of 43 lb. Ignore the weight of the rope. With what force must the man pull up on the rope in order to lift himself and the platform upward at 1.2 ft/s^2?

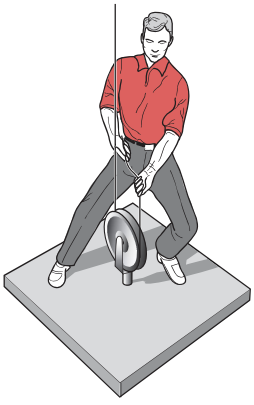


FIGURE 5-30. Exercise 8.

5-3 Frictional Forces

9. The coefficient of static friction between Teflon and scrambled eggs is about 0.04. What is the smallest angle from the horizontal that will cause the eggs to slide across the bottom of a Teflon-coated skillet?

10. Suppose that only the rear wheels of an automobile can accelerate it, and that half the total weight of the automobile is supported by those wheels. (*a*) What is the maximum acceleration attainable if the coefficient of static friction between tires and road is μ_s? (*b*) Take $\mu_s = 0.56$ and get a numerical value for this acceleration.

11. What is the greatest acceleration that can be generated by a runner if the coefficient of static friction between shoes and road is 0.95?

12. A baseball player (Fig. 5-31) with mass 79 kg, sliding into a base, is slowed by a force of friction of 470 N. What is the coefficient of kinetic friction between the player and the ground?

FIGURE 5-31. Exercise 12.

13. A horizontal bar is used to support a 75-kg object between two walls, as shown in Fig. 5-32. The equal forces F exerted by the bar against the walls can be varied by adjusting the length of the bar. Only friction between the ends of the bar and the walls supports the system. The coefficient of static friction between bar and walls is 0.41. Find the minimum value of the forces F for the system to remain at rest.

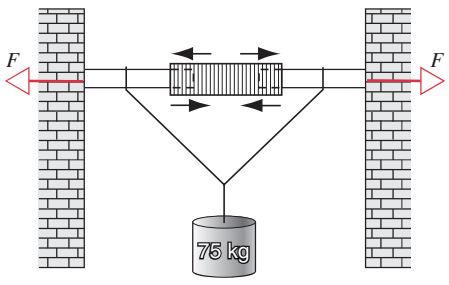


FIGURE 5-32. Exercise 13.

14. A 53-lb (= 240-N) trunk rests on the floor. The coefficient of static friction between the trunk and the floor is 0.41, while the coefficient of kinetic friction is 0.32. (*a*) What is the minimum horizontal force with which a person must push on the trunk to start it moving? (*b*) Once moving, what horizontal force must the person apply to keep the trunk moving with constant velocity? (*c*) If, instead, the person continued to push with the force used to start the motion, what would be the acceleration of the trunk?

15. The coefficient of static friction between the tires of a car and a dry road is 0.62. The mass of the car is 1500 kg. What maximum braking force is obtainable (*a*) on a level road and (*b*) on an 8.6° downgrade?

16. A house is built on the top of a hill with a 42° slope. Subsequent slumping of material on the slope surface indicates that the slope gradient should be reduced. If the coefficient of friction of soil on soil is 0.55, through what additional angle ϕ (see Fig. 5-33) should the slope surface be regraded?

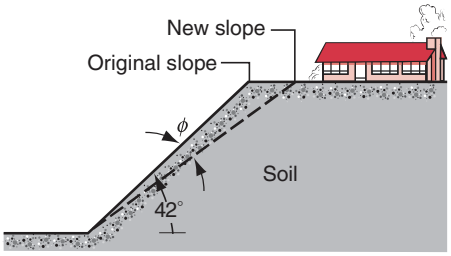


FIGURE 5-33. Exercise 16.

17. A 136-kg crate is at rest on the floor. A worker attempts to push it across the floor by applying a 412-N force horizontally. (*a*) Take the coefficient of static friction between the crate and floor to be 0.37 and show that the crate does not move. (*b*) A second worker helps by pulling up on the crate. What minimum vertical force must this worker apply so that the crate starts to move across the floor? (*c*) If the second worker applies a horizontal rather than a vertical force, what minimum force, in addition to the original 412-N force, must be exerted to get the crate started?

18. A student wants to determine the coefficients of static friction and kinetic friction between a box and a plank. She places the box on the plank and gradually raises one end of the plank. When the angle of inclination with the horizontal reaches 28.0°, the box starts to slip and slides 2.53 m down the plank in 3.92 s. Find the coefficients of friction.

19. Frictional heat generated by the moving ski is the chief factor promoting sliding in skiing. The ski sticks at the start, but once in motion will melt the snow beneath it. Waxing the ski makes it water repellent and reduces friction with the film of water. A magazine reports that a new type of plastic ski is even more water repellent and that, on a gentle 203-m slope in the Alps, a skier reduced his time from 61 to 42 s with the new skis. Assuming a 3.0° slope, compute the coefficient of kinetic friction for each case.

20. A block slides down an inclined plane of slope angle θ with constant velocity. It is then projected up the same plane with an initial speed v_0. (*a*) How far up the incline will it move before coming to rest? (*b*) Will it slide down again?

21. A piece of ice slides from rest down a rough 33.0° incline in twice the time it takes to slide down a frictionless 33.0° incline of the same length. Find the coefficient of kinetic friction between the ice and the rough incline.

22. In Fig. 5-34, *A* is a 4.4-kg block and *B* is a 2.6-kg block. The coefficients of static and kinetic friction between *A* and the table are 0.18 and 0.15. (*a*) Determine the minimum mass of the block *C* that must be placed on *A* to keep it from sliding. (*b*) Block *C* is suddenly lifted off *A*. What is the acceleration of block *A*?

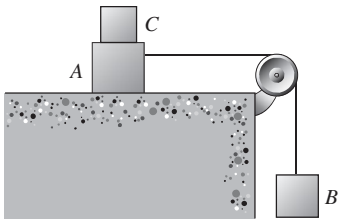


FIGURE 5-34. Exercise 22.

23. A 4.8-kg block on a 39° inclined plane is acted on by a horizontal force of 46 N (see Fig. 5-35). The coefficient of kinetic friction between block and plane is 0.33. (*a*) What is the acceleration of the block if it is moving up the plane? (*b*) With the horizontal force still acting, how far up the plane will the block go if it has an initial upward speed of 4.3 m/s? (*c*) What happens to the block after it reaches the highest point?

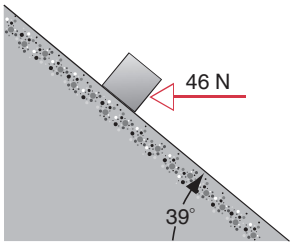


FIGURE 5-35. Exercise 23.

24. A 12-kg block of steel is at rest on a horizontal table. The coefficient of static friction between block and table is 0.52. (*a*) What is the magnitude of the horizontal force that will just start the block moving? (*b*) What is the magnitude of a force acting upward 62° from the horizontal that will just start the block moving? (*c*) If the force acts down at 62° from the horizontal, how large can it be without causing the block to move?

25. A worker drags a 150-lb crate across a floor by pulling on a rope inclined 17° above the horizontal. The coefficient of static friction is 0.52 and the coefficient of kinetic friction is 0.35. (*a*) What tension in the rope is required to start the crate moving? (*b*) What is the initial acceleration of the crate?

26. A wire will break under tensions exceeding 1.22 kN. If the wire, not necessarily horizontal, is used to drag a box across the floor, what is the greatest weight that can be moved if the coefficient of static friction is 0.35?

27. Block B in Fig. 5-36 weighs 712 N. The coefficient of static friction between block B and the table is 0.25. Find the maximum weight of block A for which block B will remain at rest.

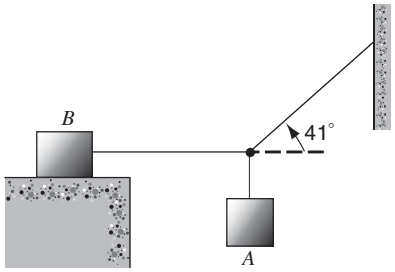


FIGURE 5-36. Exercise 27.

28. Block m_1 in Fig. 5-37 has a mass of 4.20 kg and block m_2 has a mass of 2.30 kg. The coefficient of kinetic friction between m_2 and the horizontal plane is 0.47. The inclined plane is frictionless. Find (*a*) the acceleration of the blocks and (*b*) the tension in the string.

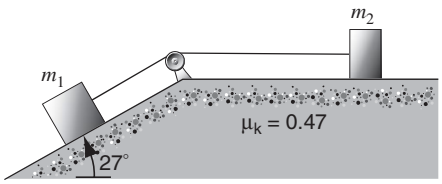


FIGURE 5-37. Exercise 28.

29. In Fig. 5-38, object B weighs 94.0 lb and object A weighs 29.0 lb. Between object B and the plane the coefficient of static friction is 0.56 and the coefficient of kinetic friction is 0.25. (*a*) Find the acceleration of the system if B is initially at rest. (*b*) Find the acceleration if B is moving up the plane. (*c*) What is the acceleration if B is moving down the plane? The plane is inclined by 42.0°.

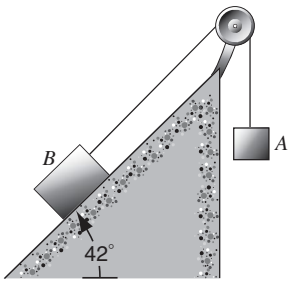


FIGURE 5-38. Exercise 29.

30. A crate slides down an inclined right-angled trough as in Fig. 5-39. The coefficient of kinetic friction between the crate and the material composing the trough is μ_k. Find the acceleration of the crate.

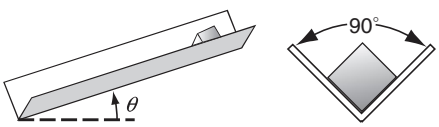


FIGURE 5-39. Exercise 30.

31. A 42-kg slab rests on a frictionless floor. A 9.7-kg block rests on top of the slab, as in Fig. 5-40. The coefficient of static friction between the block and the slab is 0.53, while the coefficient of kinetic friction is 0.38. The 9.7-kg block is acted on by a horizontal force of 110 N. What are the resulting accelerations of (*a*) the block and (*b*) the slab?

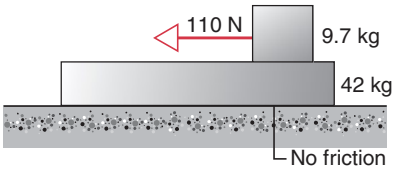


FIGURE 5-40. Exercise 31.

5-4 The Dynamics of Uniform Circular Motion

32. During an Olympic bobsled run, a European team takes a turn of radius 25 ft at a speed of 60 mi/h. What acceleration do the riders experience (*a*) in ft/s^2 and (*b*) in units of g?

33. A 2400-lb (= 10.7-kN) car traveling at 30 mi/h (= 13.4 m/s) attempts to round an unbanked curve with a radius of 200 ft (= 61.0 m). (*a*) What force of friction is required to keep the car on its circular path? (*b*) What minimum coefficient of static friction between the tires and road is required?

34. A circular curve of highway is designed for traffic moving at 60 km/h (= 37 mi/h). (*a*) If the radius of the curve is 150 m (= 490 ft), what is the correct angle of banking of the road? (*b*) If the curve were not banked, what would be the minimum coefficient of friction between tires and road that would keep traffic from skidding at this speed?

35. A conical pendulum is formed by attaching a 53-g pebble to a 1.4-m string. The pebble swings around in a circle of radius 25 cm. (*a*) What is the speed of the pebble? (*b*) What is its acceleration? (*c*) What is the tension in the string?

36. A bicyclist (Fig. 5-41) travels in a circle of radius 25 m at a constant speed of 8.7 m/s. The combined mass of the bicycle and rider is 85 kg. Calculate the force—magnitude and angle with the vertical—exerted by the road on the bicycle.

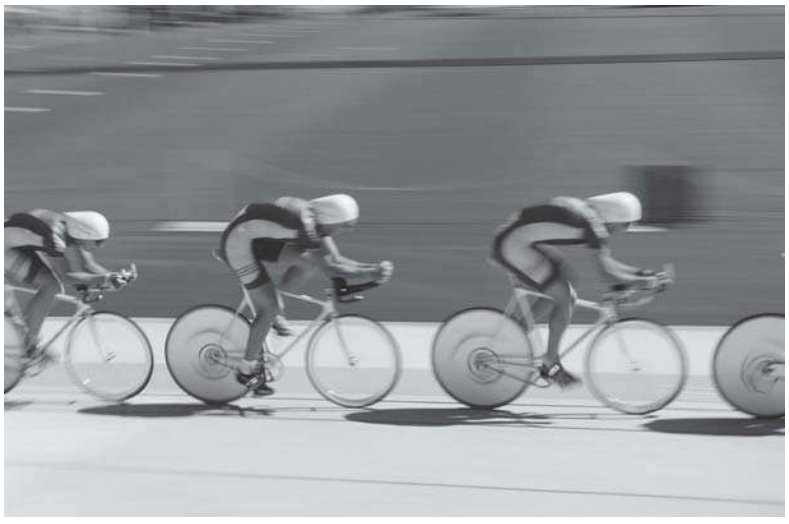


FIGURE 5-41. Exercise 36.

37. In the Bohr model of the hydrogen atom, the electron revolves in a circular orbit around the nucleus. If the radius is 5.3×10^{-11} m and the electron makes 6.6×10^{15} rev/s, find (*a*) the speed of the electron, (*b*) the acceleration of the electron, and (*c*) the force acting on the electron. (This force is the result of the attraction between the positively charged nucleus and the negatively charged electron.)

38. A child places a picnic basket on the outer rim of a merry-go-round that has a radius of 4.6 m and revolves once every 24 s. How large must the coefficient of static friction be for the basket to stay on the merry-go-round?

39. A disk of mass m on a frictionless table is attached to a hanging cylinder of mass M by a cord through a hole in the table (see Fig. 5-42). Find the speed with which the disk must move in a circle of radius r for the cylinder to stay at rest.

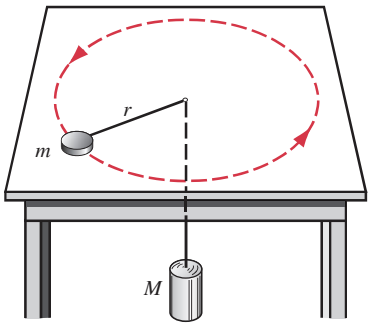


FIGURE 5-42. Exercise 39.

40. A driver's manual states that a driver traveling at 48 km/h and desiring to stop as quickly as possible travels 10 m before the foot reaches the brake. The car travels an additional 21 m before coming to rest. (*a*) What coefficient of friction is assumed in these calculations? (*b*) What is the minimum radius for turning a corner at 48 km/h without skidding?

41. A banked circular highway curve is designed for traffic moving at 95 km/h. The radius of the curve is 210 m. Traffic is moving along the highway at 52 km/h on a stormy day. (*a*) What is the minimum coefficient of friction between tires and road that will allow cars to negotiate the turn without sliding? (*b*) With this value of the coefficient of friction, what is the greatest speed at which the curve can be negotiated without sliding?

42. A 150-lb student on a steadily rotating Ferris wheel is sitting on a scale that reads 125 lb at the highest point. (*a*) What is the scale reading at the lowest point? (*b*) What would be the scale reading at the highest point if the speed of the Ferris wheel were doubled?

43. A small object is placed 13.0 cm from the center of a phonograph turntable. It is observed to remain on the table when it rotates at $33\frac{1}{3}$ rev/min but slides off when it rotates at 45.0 rev/min. Between what limits must the coefficient of static friction between the object and the surface of the turntable lie?

44. An airplane is flying in a horizontal circle at a speed of 482 km/h. The wings of the plane are tilted at 38.2° to the horizontal; see Fig. 5-43. Find the radius of the circle in which the plane is flying. Assume that the centripetal force is provided entirely by the lift force perpendicular to the wing surface.

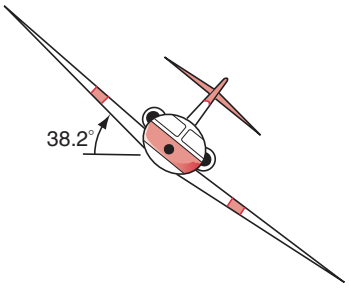


FIGURE 5-43. Exercise 44.

45. A frigate bird is soaring in a horizontal circular path. Its bank angle is estimated to be 25° and it takes 13 s for the bird to complete one circle. (*a*) How fast is the bird flying? (*b*) What is the radius of the circle? (See "The Amateur Scientist" by Jearl Walker, *Scientific American,* March 1985, p. 122.)

46. A model airplane of mass 0.75 kg is flying at constant speed in a horizontal circle at one end of a 33-m cord and at a height of 18 m. The other end of the cord is tethered to the ground. The airplane makes 4.4 rev/min and the lift is perpendicular to the unbanked wings. (*a*) What is the acceleration of the plane? (*b*) What is the tension in the cord? (*c*) What is the lift produced by the plane's wings?

47. Assume that the standard kilogram would weigh exactly 9.80 N at sea level on the equator if the Earth did not rotate. Then take into account the fact that the Earth does rotate, so that this object moves in a circle of radius 6370 km (the Earth's radius) in one day. (*a*) Determine the centripetal force needed to keep the standard kilogram moving in its circular path. (*b*) Find the force exerted by the standard kilogram on a spring balance from which it is suspended at the equator (its apparent weight).

5-5 Time-Dependent Forces

48. The position of a particle of mass 2.17 kg traveling in a straight line is given by

$$x = (0.179\ \text{m/s}^4)t^4 - (2.08\ \text{m/s}^2)t^2 + 17.1\ \text{m}.$$

Find the (*a*) velocity, (*b*) acceleration, and (*c*) force on the particle at time $t = 7.18$ s.

49. A particle of mass m is subjected to a net force $\vec{\mathbf{F}}(t)$ given by $\vec{\mathbf{F}}(t) = F_0(1 - t/T)\hat{\mathbf{i}}$; that is, $F(t)$ equals F_0 at $t = 0$ and decreases linearly to zero in time T. The particle passes the origin $x = 0$ with velocity $v_0\hat{\mathbf{i}}$. Show that at the instant $t = T$ that $F(t)$ vanishes, the speed v and distance x traveled are given by $v(T) = v_0 + a_0T/2$, and $x(T) = v_0T + a_0T^2/3$, where $a_0 = F_0/m$ is the initial acceleration. Compare these results with Eqs. 2-26 and 2-28.

5-6 Noninertial Frames and Pseudoforces

5-7 Limitations of Newton's Laws

PROBLEMS

1. A block of mass m_1 on a frictionless inclined plane making an angle θ_1 with the horizontal is connected by a cord over a small frictionless massless pulley to a second block of mass m_2 on a frictionless plane at an angle θ_2 (see Fig. 5-44). (*a*) Show that the acceleration of each block is

$$a = \frac{m_1 \sin \theta_1 - m_2 \sin \theta_2}{m_1 + m_2} g$$

and that the tension in the cord is

$$T = \frac{m_1 m_2 g}{m_1 + m_2} (\sin \theta_1 + \sin \theta_2).$$

(*b*) Evaluate the acceleration and tension for $m_1 = 3.70$ kg and $m_2 = 4.86$ kg when $\theta_1 = 28°$ and $\theta_2 = 42°$. What direction does m_1 move along the plane? (*c*) Using the values of m_1, θ_1, and θ_2 given above, for what values of m_2 does m_1 accelerate up the plane? Accelerate down the plane? Not accelerate?

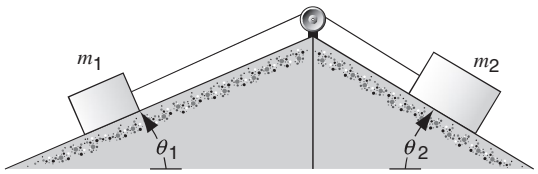


FIGURE 5-44. Problem 1.

2. Someone exerts a force F directly up on the axle of the pulley shown in Fig. 5-45. Consider the pulley and string to be massless and the bearing frictionless. Two objects, m_1 of mass 1.2 kg and m_2 of mass 1.9 kg, are attached as shown to the opposite ends of the string, which passes over the pulley. The object m_2 is in contact with the floor. (*a*) What is the largest value the force $\vec{\mathbf{F}}$ may have so that m_2 will remain at rest on the floor? (*b*) What is the tension in the string if the upward force F is 110 N? (*c*) With the tension determined in part (*b*), what is the acceleration of m_1?

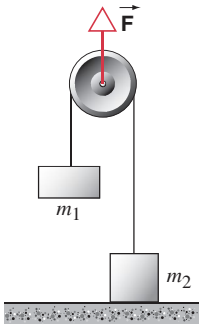


FIGURE 5-45. Problem 2.

3. Two particles, each of mass m, are connected by a light string of length $2L$, as shown in Fig. 5-46. A steady force $\vec{\mathbf{F}}$ is applied at the midpoint of the string ($x = 0$) at a right angle to the initial position of the string. Show that the acceleration of each mass in the direction at 90° to $\vec{\mathbf{F}}$ is given by

$$a_x = \frac{F}{2m} \frac{x}{(L^2 - x^2)^{1/2}}$$

in which x is the perpendicular distance of one of the particles from the line of action of $\vec{\mathbf{F}}$. Discuss the situation when $x = L$.

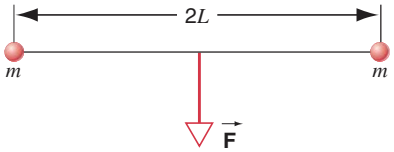


FIGURE 5-46. Problem 3.

4. A horizontal force F of 12 lb pushes a block weighing 5.0 lb against a vertical wall (Fig. 5-47). The coefficient of static friction between the wall and the block is 0.60 and the coefficient of kinetic friction is 0.40. Assume the block is not moving initially. (*a*) Will the block start moving? (*b*) What is the force exerted on the block by the wall?

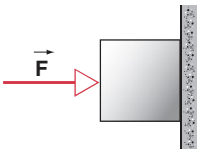


FIGURE 5-47. Problem 4.

5. A 7.96-kg block rests on a plane inclined at 22.0° to the horizontal, as shown in Fig. 5-48. The coefficient of static friction is 0.25, while the coefficient of kinetic friction is 0.15. (*a*) What is the minimum force F, parallel to the plane, that will prevent the block from slipping down the plane? (*b*) What is the minimum force F that will start the block moving up the plane? (*c*) What force F is required to move the block up the plane at constant velocity?

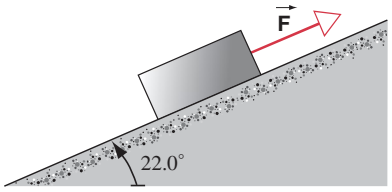


FIGURE 5-48. Problem 5.

6. A worker wishes to pile sand onto a circular area in his yard. The radius of the circle is R. No sand is to spill onto the surrounding area; see Fig. 5-49. Show that the greatest volume of sand that can be stored in this manner is $\pi \mu_s R^3/3$, where μ_s is the coefficient of static friction of sand on sand. (The volume of a cone is $Ah/3$, where A is the base area and h is the height.)

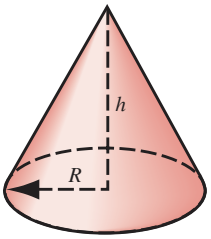


FIGURE 5-49. Problem 6.

7. The handle of a floor mop of mass m makes an angle θ with the vertical direction; see Fig. 5-50. Let μ_k be the coefficient of kinetic friction between mop and floor and μ_s the coefficient of static friction between mop and floor. Neglect the mass of the handle. (*a*) Find the magnitude of the force F directed along the handle required to slide the mop with uniform velocity across the floor. (*b*) Show that if θ is smaller than a certain angle θ_0 the mop cannot be made to slide across the floor no matter how great a force is directed along the handle. What is the angle θ_0?

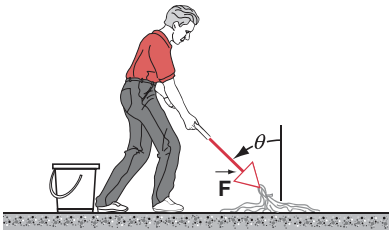


FIGURE 5-50. Problem 7.

8. Figure 5-51 shows the cross section of a road cut into the side of a mountain. The solid line AA' represents a weak bedding plane along which sliding is possible. The block B directly above the highway is separated from uphill rock by a large crack (called a *joint*), so that only the force of friction between the block and the likely surface of failure prevent sliding. The mass of the block is 1.8×10^7 kg, the dip angle of the failure plane is 24°, and the coefficient of static friction between block and plane is 0.63. (*a*) Show that the block will not slide. (*b*) Water seeps into the joint, exerting a hydrostatic force F parallel to the incline on the block. What minimum value of F will trigger a slide?

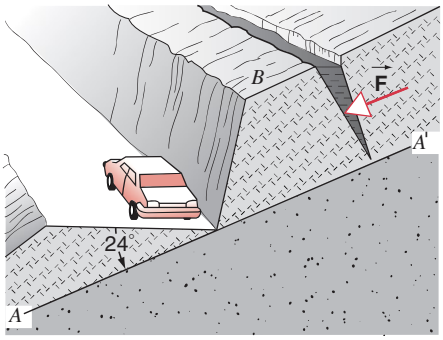


FIGURE 5-51. Problem 8.

9. The two blocks, $m = 16$ kg and $M = 88$ kg, shown in Fig. 5-52 are free to move. The coefficient of static friction between the blocks is $\mu_s = 0.38$, but the surface beneath M is frictionless. What is the minimum horizontal force F required to hold m against M?

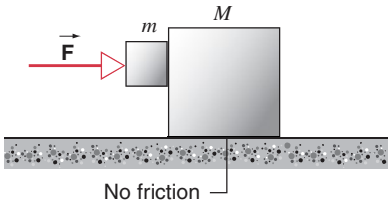


FIGURE 5-52. Problem 9.

10. Two objects, with masses $m_1 = 1.65$ kg and $m_2 = 3.22$ kg, attached by a massless rod parallel to the incline on which both slide, as shown in Fig. 5-53, travel down the plane with m_1 trailing m_2. The angle of the incline is $\theta = 29.5°$. The coefficient of kinetic friction between m_1 and the incline is $\mu_1 = 0.226$; between m_2 and the incline the corresponding coefficient is $\mu_2 = 0.127$. Compute (*a*) the common acceleration of the two objects and (*b*) the tension in the rod. (*c*) What are the answers to (*a*) and (*b*) if m_2 trails m_1?

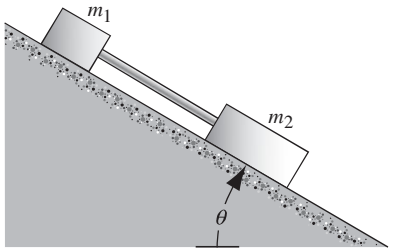


FIGURE 5-53. Problem 10.

11. A massless rope is tossed over a wooden dowel of radius r in order to lift a heavy object of weight W off of the floor, as shown in Fig. 5-54. The coefficient of sliding friction between the rope and the dowel is μ. Show that the minimum downward pull on the rope necessary to lift the object is

$$F_{\text{down}} = We^{\pi\mu}.$$

(Hint: This problem requires techniques from integral calculus.)

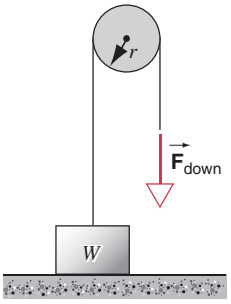


FIGURE 5-54. Problem 11.

12. A 4.40-kg block is put on top of a 5.50-kg block. In order to cause the top block to slip on the bottom one, held fixed, a horizontal force of 12.0 N must be applied to the top block. The assembly of blocks is now placed on a horizontal, frictionless table; see Fig. 5-55. Find (*a*) the maximum horizontal force F that can be applied to the lower block so that the blocks will move together, (*b*) the resulting acceleration of the blocks, and (*c*) the coefficient of static friction between the blocks.

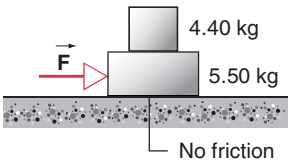


FIGURE 5-55. Problem 12.

13. You are driving a car at a speed of 85 km/h when you notice a barrier across the road 62 m ahead. (*a*) What is the minimum coefficient of static friction between tires and road that will allow you to stop without striking the barrier? (*b*) Suppose that you are driving at 85 km/h on a large empty parking lot. What is the minimum coefficient of static friction that would allow you to turn the car in a 62-m radius circle and, in this way, avoid collision with a wall 62 m ahead?

14. A car moves at a constant speed on a straight but hilly road. One section has a crest and dip of the same 250-m radius; see Fig. 5-56. (*a*) As the car passes over the crest, the normal force on the car is one-half the 16-kN weight of the car. What will be the normal force on the car as it passes through the bottom of the dip? (*b*) What is the greatest speed at which the car can move without leaving the road at the top of the hill? (*c*) Moving at the speed found in (*b*), what will be the normal force on the car as it moves through the bottom of the dip?

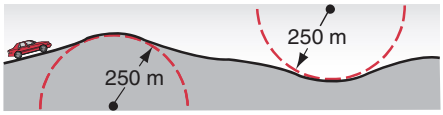


FIGURE 5-56. Problem 14.

15. A small coin is placed on a flat, horizontal turntable. The turntable is observed to make exactly three revolutions in 3.3 s. (*a*) What is the speed of the coin when it rides without slipping at a distance of 5.2 cm from the center of the turntable? (*b*) What is the acceleration (magnitude and direction) of the coin in part (*a*)? (*c*) What is the force of friction acting on the coin in part (*a*) if the coin has a mass of 1.7 g? (*d*) What is the coefficient of static friction between the coin and the turntable if the coin is observed to slide off the turntable when it is more than 12 cm from the center of the turntable?

16. A certain string can withstand a maximum tension of 9.2 lb without breaking. A child ties a 0.82-lb stone to one end and, holding the other end, whirls the stone in a vertical circle of radius 2.9 ft, slowly increasing the speed until the string breaks. (*a*) Where is the stone on its path when the string breaks? (*b*) What is the speed of the stone as the string breaks?

17. A 1.34-kg ball is attached to a rigid vertical rod by means of two massless strings each 1.70 m long. The strings are attached to the rod at points 1.70 m apart. The system is rotating about the axis of the rod, both strings being taut and forming an equilateral triangle with the rod, as shown in Fig. 5-57. The tension in the upper string is 35.0 N. (*a*) Find the tension in the lower string. (*b*) Calculate the net force on the ball at the instant shown in the figure. (*c*) What is the speed of the ball?

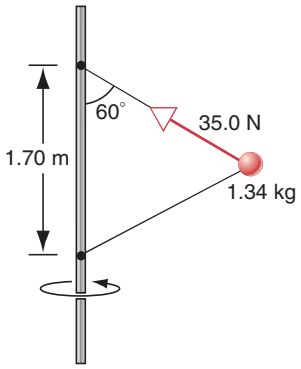


FIGURE 5-57. Problem 17.

18. A very small cube of mass m is placed on the inside of a funnel (see Fig. 5-58) rotating about a vertical axis at a constant rate of ω revolutions per second. The wall of the funnel makes an angle θ with the horizontal. The coefficient of static friction between cube and funnel is μ_s and the center of the cube is at a distance r from the axis of rotation. Find the (*a*) largest and (*b*) smallest values of ω for which the cube will not move with respect to the funnel.

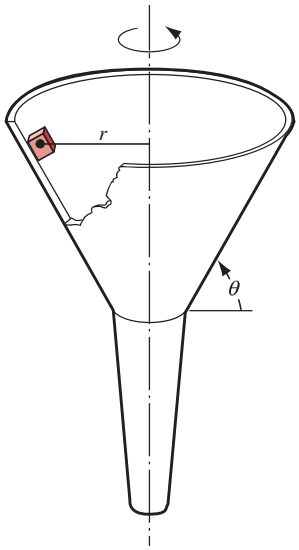


FIGURE 5-58. Problem 18.

19. Because of the rotation of the Earth, a plumb bob may not hang exactly along the direction of the Earth's gravitational force on the plumb bob but may deviate slightly from that di-

rection. (*a*) Show that the deflection θ in radians at a point at latitude L is given by

$$\theta = \left(\frac{2\pi^2 R}{gT^2}\right) \sin 2L,$$

where R is the radius of the Earth and T is the period of the Earth's rotation. (*b*) At what latitude is the deflection a maximum? How much is this deflection? (*c*) What is the deflection at the poles? At the equator?

20. A particle of mass m sits at rest at $x = 0$. At time $t = 0$ a force given by $F = F_0 e^{-t/T}$ is applied in the $+x$ direction; F_0 and T are constants. When $t = T$ the force is removed. At this instant when the force is removed, (*a*) what is the speed of the particle and (*b*) where is it?

COMPUTER PROBLEMS

1. An automobile is moving at a constant speed while pulling a block of wood of mass $m = 200$ kg with an elastic cord. The force exerted on the block of wood by the cord depends on the length of the cord, and is given by $F = 400(l - 10)$ where F is measured in newtons when l is measured in meters. This force, however, is 0 if $l < 10$ m. The coefficient of static friction between the block and ground is $\mu_s = 0.60$, while the coefficient of kinetic friction is $\mu_k = 0.50$. Set up a computer program to numerically evaluate the motion of the block under the following scenarios: (*a*) Assume the block is originally at rest and the automobile is originally 10 m distant and moving away from the block with a constant speed of 5 m/s. (*b*) Assume the block is originally at rest and the automobile is originally 10 m distant and moving away from the block with a constant speed of 20 m/s.

2. Starting from rest a person pushes a 95-kg crate across a rough floor with a force given by $F = 200e^{-0.15t}$ where F is in newtons and t is in seconds. The force decreases exponentially because the person tires. As long as the crate is moving a constant frictional force of 80 N opposes the motion. (*a*) How long after starting does the crate stop? (*b*) How far does it go? (*c*) How accurate are your results? (Try using the Euler method with an initial time interval of $\Delta t = 0.01$ s. Repeat the process, but use a time interval of $\Delta t = 0.001$ s. Compare the results to get an estimate of your accuracy.)